MX SP 13 b A device playing a signal given by $x = \sqrt{2}\sin t + \cos t$ produces distortion whenever $|x| \geq 1.5$.
For what fraction of the time will the device produce distortion if the signal is played continuously? **4**

Rewrite $\sqrt{2}\sin t + \cos t$ as $\cos t + \sqrt{2}\sin t$ and express in the form $R\cos(t - \alpha)$:

$r = \sqrt{1^2 + (\sqrt{2})^2} = \sqrt{3}$, and $\tan\alpha = \sqrt{2}$.

As $|x| \geq 1.5$ then consider $|\sqrt{3}\cos(t - \alpha)| \geq \frac{3}{2}$

$$|\cos(t - \alpha)| \geq \frac{3}{2\sqrt{3}}$$

$$|\cos(t - \alpha)| \geq \frac{\sqrt{3}}{2}$$

Now in the domain $[0, \frac{\pi}{2}]$, $\cos(t - \alpha) \geq \frac{\sqrt{3}}{2}$ when $[0, \frac{\pi}{6}]$, which is one-third of the time.

This fraction is consistent for all multiples of $\frac{\pi}{2}$s, so the fraction is $\frac{1}{3}$.