



- MX 13** (i) Prove the trigonometric identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **3**
- SP c** (ii) Hence find expressions for the exact values of the solutions to the equation **4**
- $$8x^3 - 6x = 1.$$

(i)

$$\begin{aligned} \text{LHS} &= \cos 3\theta \\ &= \cos(2\theta + \theta) \\ &= \cos 2\theta \cos \theta - \sin 2\theta \sin \theta \\ &= (2\cos^2\theta - 1)\cos\theta - 2\sin\theta \cos\theta \sin\theta \\ &= 2\cos^3\theta - \cos\theta - 2\cos\theta \sin^2\theta \\ &= 2\cos^3\theta - \cos\theta - 2\cos\theta(1 - \cos^2\theta) \\ &= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta \\ &= 4\cos^3\theta - 3\cos\theta \\ &= \text{RHS} \end{aligned}$$

(ii) From (i), $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$$2\cos 3\theta = 8\cos^3\theta - 6\cos\theta$$

$$8\cos^3\theta - 6\cos\theta = 2\cos 3\theta$$

Let $x = \cos\theta$:

$$x^3 - 6x = 2\cos 3\theta = 1$$

Hence, need to solve $2\cos 3\theta = 1$:

$$2\cos 3\theta = 1$$

$$\cos 3\theta = \frac{1}{2}$$

$$3\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}, \dots$$

$$\theta = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}, \dots$$

Hence, unique solutions are $x = \cos \frac{\pi}{9}$, $\cos \frac{5\pi}{9}$ and $\cos \frac{7\pi}{9}$ (as others provide same solutions)

* These solutions have been provided by [projectmaths](http://projectmaths.com.au) and are not supplied or endorsed by NESA.

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