**MX SP** **13 c**

(i) Prove the trigonometric identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **3**

(ii) Hence find expressions for the exact values of the solutions to the equation $8x^3 - 6x = 1$. **4**

(i)

$$\begin{aligned}
\text{LHS} &= \cos 3\theta \\
&= \cos(2\theta + \theta) \\
&= \cos 2\theta \cos\theta - \sin 2\theta \sin\theta \\
&= (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\sin\theta \\
&= 2\cos^3\theta - \cos\theta - 2\cos\theta\sin^2\theta \\
&= 2\cos^3\theta - \cos\theta - 2\cos\theta(1 - \cos^2\theta) \\
&= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta \\
&= 4\cos^3\theta - 3\cos\theta \\
&= \text{RHS}
\end{aligned}$$

(ii) From (i), $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$$2\cos 3\theta = 8\cos^3\theta - 6\cos\theta$$

$$8\cos^3\theta - 6\cos\theta = 2\cos 3\theta$$

Let $x = \cos\theta$:

$x^3 - 6x = 2\cos 3\theta = 1$

Hence, need to solve $2\cos 3\theta = 1$:

$$2\cos 3\theta = 1$$

$$\cos 3\theta = \frac{1}{2}$$

$$3\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}, \ldots$$

$$\theta = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}, \ldots$$

Hence, unique solutions are $x = \cos\frac{\pi}{9}$, $\cos\frac{5\pi}{9}$ and $\cos\frac{7\pi}{9}$ (as others provide same solutions)

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by NESA.