MX SQ 14 a

(i) Sketch the graph of $y = x\cos x$ for $-\pi \le x \le \pi$ and hence explain why **3**

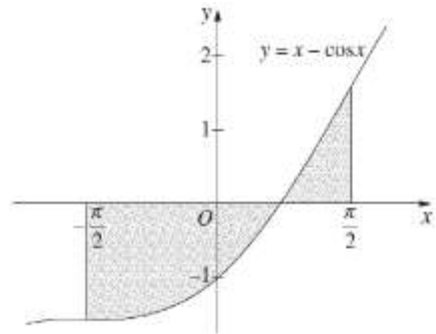
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x\,dx = 0.$$

(ii) Consider the volume of the solid of revolution produced by rotating about the x-axis the shaded region between the graph of **3**

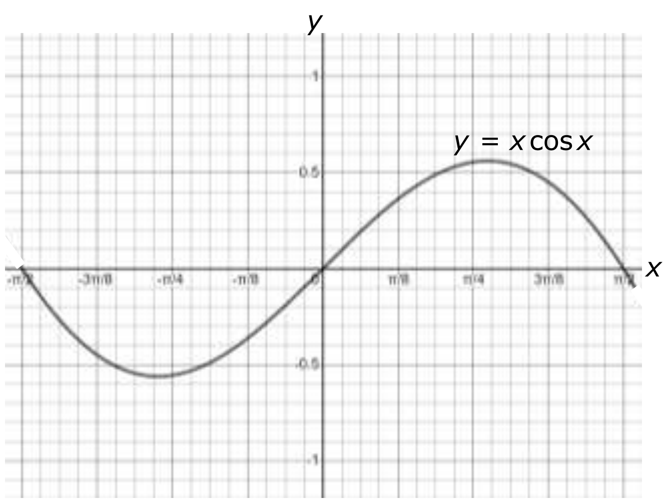
$y = x - \cos x$, the x-axis and the lines

$x = -\dfrac{\pi}{2}$ and $x = -\dfrac{\pi}{2}$.

Using the results of part (a), or otherwise, find the volume of the solid.



(i)



As $y = x\cos x$ is an odd function then

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x\,dx = 0.$$

(ii) Volume $= \pi \displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x - \cos x)^2\,dx$

$$= \pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(x^2 - 2x\cos x + \cos^2 x\right) dx$$

$$= \pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(x^2 - 2x\cos x + \frac{1}{2}(1 + \cos 2x)\right) dx$$

$$= \pi[\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2\,dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2x\cos x\,dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}(1 + \cos 2x)\,dx$$

$$= \pi[\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2\,dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}(1 + \cos 2x)\,dx$$

$$= \frac{\pi}{3}\left[x^3\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{\pi}{2}\left[x + \frac{1}{2}\sin 2x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{\pi}{3}\left[(\frac{\pi}{2})^3 - (-\frac{\pi}{2})^3\right]$$

$$+ \frac{\pi}{2}\left[\frac{\pi}{2} + \frac{1}{2}\sin 2(\frac{\pi}{2}) - (-\frac{\pi}{2} + \frac{1}{2}\sin 2(-\frac{\pi}{2}))\right]$$

	$= \frac{\pi}{3}(\frac{\pi^3}{4}) + \frac{\pi}{2}(\pi)$ $= \frac{\pi^4 + 6\pi^2}{12}$