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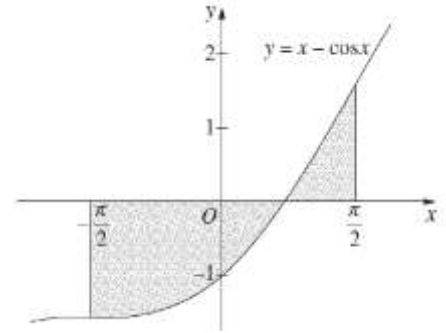
MX 14 (i) Sketch the graph of $y = x \cos x$ for $-\pi \leq x \leq \pi$ and hence explain why **3**
SQ a

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos x \, dx = 0.$$

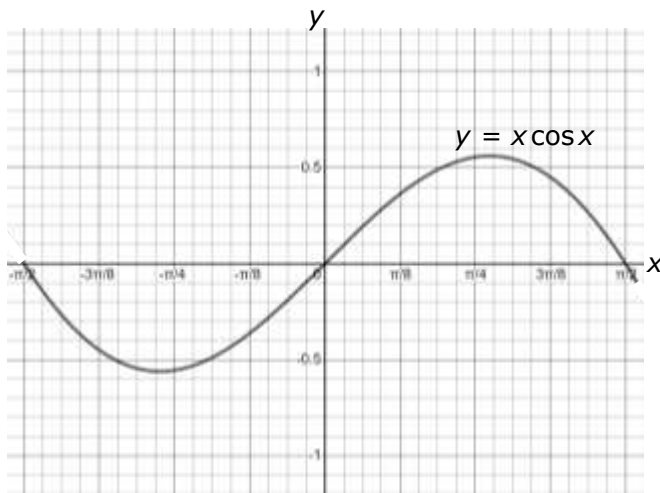
(ii) Consider the volume of the solid of revolution produced by rotating about the x -axis the shaded region between the graph of $y = x - \cos x$, the x -axis and the lines

$$x = -\frac{\pi}{2} \text{ and } x = \frac{\pi}{2}.$$

Using the results of part (a), or otherwise, find the volume of the solid.



(i)



As $y = x \cos x$ is an odd function then

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos x \, dx = 0.$$

$$\begin{aligned} \text{(ii) Volume} &= \pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x - \cos x)^2 \, dx \\ &= \pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^2 - 2x \cos x + \cos^2 x) \, dx \\ &= \pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(x^2 - 2x \cos x + \frac{1}{2}(1 + \cos 2x) \right) \, dx \\ &= \pi \left[\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \, dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2x \cos x \, dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}(1 + \cos 2x) \, dx \right] \\ &= \pi \left[\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \, dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}(1 + \cos 2x) \, dx \right] \\ &= \frac{\pi}{3} \left[x^3 \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \frac{\pi}{2} \left[x + \frac{1}{2} \sin 2x \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\ &= \frac{\pi}{3} \left[\left(\frac{\pi}{2}\right)^3 - \left(-\frac{\pi}{2}\right)^3 \right] \\ &\quad + \frac{\pi}{2} \left[\frac{\pi}{2} + \frac{1}{2} \sin 2\left(\frac{\pi}{2}\right) - \left(-\frac{\pi}{2} + \frac{1}{2} \sin 2\left(-\frac{\pi}{2}\right)\right) \right] \end{aligned}$$



$$\begin{aligned} &= \frac{\pi}{3} \left(\frac{\pi^3}{4} \right) + \frac{\pi}{2} (\pi) \\ &= \frac{\pi^4 + 6\pi^2}{12} \end{aligned}$$

* These solutions have been provided by [projectmaths](http://projectmaths.com.au) and are not supplied or endorsed by NESA.