MX SP 14 b The population of foxes on an island is modelled by the logistic equation $\frac{dy}{dt} = y(1 - y)$, where y is the fraction of the island's carrying capacity reached after t years.

At time $t = 0$, the population of foxes is estimated to be one-quarter of the island's carrying capacity.

(i) Use the substitution $y = \frac{1}{1-w}$ to transform the logistic equation to $\frac{dw}{dt} = -w$. **2**

(ii) Using the solution of $\frac{dw}{dt} = -w$, find the solution of the logistic equation for y satisfying the initial conditions. **2**

(iii) How long will it take for the fox population to reach three-quarters of the island's carrying capacity? **2**

(i) $\frac{dy}{dt} = y(1 - y)$

$= \frac{1}{1-w}(1 - \frac{1}{1-w})$

$= \frac{1}{1-w}(\frac{1-w-1}{1-w})$

$= \frac{1}{1-w}(\frac{-w}{1-w})$

$= \frac{-w}{(1-w)^2}$

Also, $y = \frac{1}{1-w}$

$= (1 - w)^{-1}$

$\frac{dy}{dw} = -1(1 - w)^{-2}.-1$

$= \frac{1}{(1-w)^2}$

As $\frac{dy}{dt} = \frac{dy}{dw} \times \frac{dw}{dt}$,

$\frac{-w}{(1-w)^2} = \frac{1}{(1-w)^2} \times \frac{dw}{dt}$

$\frac{dw}{dt} = -w$

(ii) $\frac{dw}{dt} = -w$

$\int \frac{1}{w}\, dw = \int -1\, dt$

$\ln w = -t + C$

$w = e^{-t+C}$

$w = Ae^{-t}$ *, where $A = e^c$

Now, substitute $y = \frac{1}{4}$ into $y = \frac{1}{1-w}$:

$\frac{1}{4} = \frac{1}{1-w}$

$\therefore\ w = -3$

Hence substitute $w = -3$ and $t = 0$ into *:

$-3 = Ae^0$

$A = -3$

$\therefore\ w = -3e^{-t}$

Hence, $y = \frac{1}{1+3e^{-t}}$.

(iii) $\frac{3}{4} = \frac{1}{1+3e^{-t}}$

$1 + 3e^{-t} = \frac{4}{3}$

$3e^{-t} = \frac{1}{3}$

$e^{-t} = \frac{1}{9}$

$-t = \ln \frac{1}{9}$

$t = \ln 9$ $\therefore$ will take $\ln 9$ days

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by NESA.