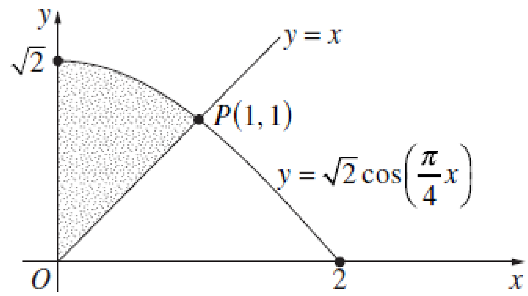
Want more revision exercises? Get **MathsFit HSC Mathematics** for $2.95/topic

2016 **13 d** The curve $y = \sqrt{2}\cos\left(\frac{\pi}{4}x\right)$ meets the line $y = x$ at $P(1, 1)$, as shown in the diagram. **3**

Find the exact value of the shaded area.



$$\text{Shaded area} = \int_0^1 \sqrt{2}\cos\left(\frac{\pi}{4}x\right)\,dx - \frac{1}{2} \times 1 \times 1$$

$$= \left[\frac{4\sqrt{2}}{\pi}\sin\frac{\pi}{4}x\right]_0^1 - \frac{1}{2}$$

$$= \frac{4\sqrt{2}}{\pi}\left[\sin\frac{\pi}{4} - \sin 0\right] - \frac{1}{2}$$

$$= \frac{4\sqrt{2}}{\pi}\left[\frac{1}{\sqrt{2}} - 0\right] - \frac{1}{2}$$

$$= \frac{4}{\pi} - \frac{1}{2}$$

$$= \frac{8-\pi}{2\pi} \qquad \therefore \text{ the area is } \frac{8-\pi}{2\pi} \text{ units}^2$$

State Mean: **1.77**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

BOSTES: Notes from the Marking Centre

Most responses included finding two areas by integration then subtracting them to find the shaded area. Common problems were:

- using incorrect boundary values for either the area under the cosine curve or the area under the line $y = x$. This included using the y-values instead of the x-values.
- not using or reading the Reference Sheet correctly, eg, having an incorrect sign when finding the primitive for the cosine function.
- when integrating $\cos(\frac{\pi}{4}x)$, multiplying the primitive by $\frac{\pi}{4}$ instead of $\frac{4}{\pi}$
- changing $\sqrt{2}$ to $2^{\frac{1}{2}}$ and then integrating that expression
- using $A = \frac{1}{2}r^2\theta$ to calculate the area as though the shaded area was a sector
- not using the fact that $\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ and that the question asked for an exact value.