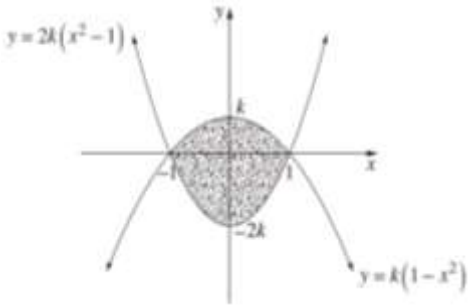
2017 14 d The shaded region shown is enclosed by two parabolas, each with x-intercepts at $x = -1$ and $x = 1$.
The parabolas have equations $y = 2k(x^2 - 1)$ and $y = k(1 - x^2)$, where $k > 0$.
Given that the area of the shaded region is 8, find the value of k.



3

$$A = 2\int_0^1 k(1 - x^2) - 2k(x^2 - 1)\,dx = 8$$

$$2\int_0^1 k - kx^2 - 2kx^2 + 2k\,dx = 8$$

$$\int_0^1 3k - 3kx^2\,dx = 4$$

$$\left[3kx - kx^3\right]_0^1 = 4$$

$$3k(1) - k(1)^3 - 0 = 4$$

$$3k - k = 4$$

$$2k = 4$$

$$k = 2$$

State Mean: **1.68**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by BOSTES.

BOSTES: Notes from the Marking Centre

Common problems were:

- making algebraic errors when subtracting the functions
- finding an incorrect primitive
- substituting into k instead of x in the primitive function when evaluating the limits.