2017 15 b Anita opens a savings account. At the start of each month she deposits $\$X$ into the savings account. At the end of the month, after interest is added into the savings account, the bank withdraws $2500 from the savings account as a loan repayment. Let M_n be the amount in the savings account after the n^{th} withdrawal. The savings account pays interest at 4.2% per annum compounded monthly.

(i) Show that after the second withdrawal the amount in the savings account is given by $M_2 = X(1.0035^2 + 1.0035) - 2500(1.0035 + 1)$. **2**

(ii) Find the value of X so that the amount in the savings account is $80 000 after the last withdrawal of the fourth year. **3**

(i) 4.2% pa = 0.35% per month = 0.0035 per month, 4 years = 48 months.

$$M_1 = X \times 1.0035 - 2500$$

$$M_2 = M_1 \times 1.0035 + X \times 1.0035 - 2500$$

$$= (X \times 1.0035 - 2500) \times 1.0035 + X \times 1.0035 - 2500$$

$$= X \times 1.0035^2 - 2500 \times 1.0035 + X \times 1.0035 - 2500$$

$$= X(1.0035^2 + 1.0035) - 2500(1.0035 + 1)$$

State Mean: **1.09**

(ii) $$M_{48} = X(1.0035^{48} + 1.0035^{47} + \ldots + 1.0035) - 2500(1.0035^{47} + 1.0035^{46} + \ldots + 1)$$

$$= X(1.0035 + 1.0035^2 + \ldots + 1.0035^{48}) - 2500(1 + 1.0035^2 + \ldots + 1.0035^{47})$$

Using geometric series and $S_n = \dfrac{a(r^n - 1)}{r - 1}$:

$$M_{48} = X\left[\frac{1.0035(1.0035^{48} - 1)}{1.0035 - 1}\right] - 2500\left[\frac{1(1.0035^{48} - 1)}{1.0035 - 1}\right] = 80\,000$$

$$52.351X - 130\,421 = 80\,000$$

$$X = 4019 \text{ (nearest whole)}$$

State Mean: **1.44**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by NESA.

NESA: Notes from the Marking Centre

(i) In most responses, students were able to find the correct expression for M_1 and many correctly showed how to obtain the expression for M_2.

Common problems were:

- attempting to work backwards from the given expression for M_2
- omitting brackets or not closing brackets
- not including a deposit of $\$X$ at the start of each month
- writing the given statement for M_2 without any working
- not checking their answer for (b)(i) was the same as the one required before continuing onto (b)(ii).

(ii) problems were:

- not using the appropriate value for n (48 months)

- using incorrect values for a and n for each series
- incorrectly applying the S_n formula for a geometric series.