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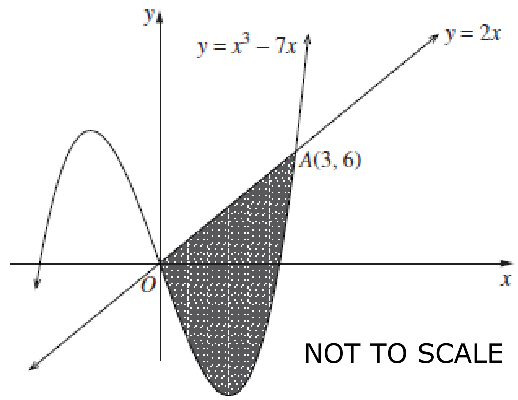
2018 15 c The shaded region is enclosed by the curve $y = x^3 - 7x$ and the line $y = 2x$ as shown in the diagram. The line $y = 2x$ meets the curve $y = x^3 - 7x$ at $O(0, 0)$ and $A(3, 6)$. DO NOT prove this.



(i) Use integration to find the area of the shaded region. **2**

(ii) Verify that one application of Simpson's rule gives the exact area of the shaded region. **2**

The point P is chosen on the curve $y = x^3 - 7x$ so that the tangent at P is parallel to the line $y = 2x$ and the x-coordinate of P is positive.

(iii) Show that the coordinates of P are $(\sqrt{3}, -4\sqrt{3})$. **2**

(iv) Find the area of ΔOAP. **2**

(i) Area $= \int_0^3 (2x - (x^3 - 7x))\ dx$

$= \int_0^3 (9x - x^3)\ dx$

$= \left[\frac{9x^2}{2} - \frac{x^4}{4}\right]_0^3$

$= \frac{9(3)^2}{2} - \frac{3^4}{4} - 0$

$= 20.25$ $\therefore$ the area is 20.25 u^2.

(ii) Using $f(x) = 9x - x^3$,

$\int_0^3 (9x - x^3)\,dx \approx \frac{b-a}{6}[f(a) + 4f(\frac{a+b}{2}) + f(b)]$

$= \frac{3-0}{6}[f(0) + 4f(1.5) + f(3)]$

$= \frac{1}{2}[0 + 4(10.125) + 0]$

$= \frac{1}{2}[0 + 4(10.125) + 0]$

$= 20.25$

$\therefore$ the area is 20.25 u^2.

(iii) $y = x^3 - 7x$

$y' = 3x^2 - 7 = 2$

$3x^2 = 9$

$x^2 = 3$

$x = \sqrt{3}$ (as $x > 0$)

$y(\sqrt{3}) = (\sqrt{3})^3 - 7(\sqrt{3})$

$= 3\sqrt{3} - 7\sqrt{3}$

$= -4\sqrt{3}$ $\therefore P(\sqrt{3}, -4\sqrt{3})$

(iv) $OA = \sqrt{3^2 + 6^2}$

$= \sqrt{45}$

$= 3\sqrt{5}$

Perp. distance from $(\sqrt{3}, -4\sqrt{3})$ to $2x - y = 0$:

$d = \frac{|2(\sqrt{3}) - 1(-4\sqrt{3}) + 0|}{\sqrt{2^2 + 1^2}}$

$= \frac{6\sqrt{3}}{\sqrt{5}}$

Area $= \frac{1}{2} \times 3\sqrt{5} \times \frac{6\sqrt{3}}{\sqrt{5}}$

$= 9\sqrt{3}$ $\therefore$ area is $9\sqrt{3}$ u^2.

State Mean:
1.27
0.9
1.09
0.54

NESA: Marking Feedback

Skills addressed:

- knowing the required area could be represented by A = $\int_a^b (f(x) - g(x))\ dx$ where a and b are x-values
- simplifying $f(x) - g(x)$ before finding the primitive function and calculating the definite integral
- using the correct formula for Simpson's rule and understanding that three function values are required for one application of the rule
- using their simplified function from (ci) to calculate their function values
- using a table to show their three functions values
- equating the gradient function for the tangent at P and the gradient of the line $y = 2x$ and solving for x
- finding the y-coordinate by substitution
- understanding that the triangle is not right angled
- using the formula for perpendicular distance provided in the Reference Sheet

Areas for students to improve include:

- writing correct statements, involving definite integrals, to use in area problems
- applying absolute value of functions correctly
- using the formula from the reference sheet and understanding the meaning of $\frac{b-a}{6}$
- finding the correct function to use in Simpson's rule
- showing substitutions used to find the y-coordinate
- performing calculations with surds
- understanding that the equation of the line used in the perpendicular distance formula needs to be in general form
- finding the correct angle at the origin to use in the sine rule