2018 16c Kara deposits an amount of \$300 000 into an account which pays compound interest of 4% per annum, added to the account at the end of each year. Immediately after the interest is added, Kara makes a withdrawal for expenses for the coming year. The first withdrawal is \$$P$. Each subsequent withdrawal is 5% greater than the previous one. Let \$$A_n$ be the amount in the account after the nth withdrawal.

(i) Show that $A_2 = 300\,000(1.04)^2 - P[(1.04) + (1.05)]$ **1**

(ii) Show that $A_3 = 300\,000(1.04)^3 - P[(1.04)^2 + (1.04)(1.05) + (1.05)^2]$. **1**

(i) Show that there will be money in the account when $\left(\frac{105}{104}\right)^n < 1 + \frac{3000}{P}$. **3**

(i) $A_1 = 300\,000 \times 1.04 - P$

$A_2 = (300\,000 \times 1.04 - P) \times 1.04 - P \times 1.05$

$= 300\,000 \times 1.04^2 - P(1.04 + 1.05)$

(ii) $A_3 = 300\,000 \times 1.04^2 - P(1.04 + 1.05) \times 1.04 - P \times 1.05^2$

$= 300\,000 \times 1.04^3 - P(1.04^2 + 1.04 \times 1.05 + 1.05^2)$

(iii) $A_n = 300\,000 \times 1.04^n - P(1.04^{n-1} + 1.04^{n-2} \times 1.05 + \ldots + 1.04 \times 1.05^{n-2} + 1.05^{n-1}))$

A geometric series with $a = 1.04^{n-1}$, $r = \frac{1.05}{1.04} = \frac{105}{104}$, $n = n$:

$$A_n = 300\,000 \times 1.04^n - P\left[\frac{1.04^{n-1}\left((\frac{105}{104})^n - 1\right)}{\frac{105}{104} - 1}\right] > 0$$

$$300\,000 \times 1.04^n - P\left[\frac{1.04^{n-1}\left((\frac{105}{104})^n - 1\right)}{\frac{1}{104}}\right] > 0$$

$$300\,000 \times 1.04^n - 104P[1.04^{n-1}\left((\frac{105}{104})^n - 1\right)] > 0$$

$$3000 \times 1.04^n - 1.04P[1.04^{n-1}\left((\frac{105}{104})^n - 1\right)] > 0$$

$$3000 \times 1.04^n - P[1.04^n\left((\frac{105}{104})^n - 1\right)] > 0$$

$$3000 - P\left((\frac{105}{104})^n - 1\right) > 0$$

$$\frac{3000}{P} - (\frac{105}{104})^n + 1 > 0 \qquad \therefore \quad (\frac{105}{104})^n < 1 + \frac{3000}{P}$$

State Mean:
0.47
0.3
0.23

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by NESA.

NESA: Marking Feedback

Skills addressed:

- having a clear understanding of the steps required to 'show' a result
- presenting all work clear sequential steps
- providing a detailed progression from A_1 through to A_2
- responding to the direction 'show that' and providing a detailed progression from A_2 to A_3
- being able to achieve an expression for A_n
- using the sum of a geometric progression formula to arrive at the given result
- demonstrating a high degree of accuracy and skill in algebraic manipulation

Areas for students to improve include:

- using brackets correctly
- remembering to increase and subtract the withdrawal, that is, using $A_2 = A_1(1.04) – P(1.05)$
- knowing that the third withdrawal was $P(1.05)^2$ and using $A_3 = A_2(1.04) – P(1.05)^2$
- using patterns to obtain an expression for A_n
- using the correct values for the first term and common ratio when finding the sum of the geometric progression