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19 M **16 a** A person wins $1 000 000 in a competition and decides to invest this money in an account that earns interest at 6% per annum compounded quarterly. The person decides to withdraw $80 000 from this account at the end of every fourth quarter.

Let A be the amount remaining in the account after the nth withdrawal.

(i) Show that the amount remaining in the account after the withdrawal at the end of the eighth quarter is $A_2 = 1\,000\,000 \times 1.015^8 - 80\,000(1 + 1.015^4)$. **2**

(ii) For how many years can the full amount of $80 000 be withdrawn? **3**

(i) 6% per annum = 1.5% per quarter

$A_1 = 1\,000\,000 \times 1.015^4 - 80\,000$ ✔

$A_2 = (1\,000\,000 \times 1.015^4 - 80\,000) \times 1.015^4 - 80\,000$

$= 1\,000\,000 \times 1.015^8 - 80\,000 \times 1.015^4 - 80\,000$

$= 1\,000\,000 \times 1.015^8 - 80\,000(1 + 1.015^4)$ ✔

State Mean: **1.08/2**

(ii) $A_n = 1\,000\,000 \times 1.015^{4n} - 80\,000(1 + 1.015^4 + 1.015^8 + \ldots + 1.015^{4(n-1)})$

Consider the series with $a = 1$, $r = 1.015^4$, $n = n$:

$$A_n = 1\,000\,000 \times 1.015^{4n} - 80\,000\left[\frac{1((1.015^4)^n - 1)}{1.015^4 - 1}\right]$$ ✔

$$= 1\,000\,000 \times 1.015^{4n} - 1\,303\,706(1.015^{4n} - 1) \le 80\,000$$ ✔

$$1\,000\,000 \times 1.015^{4n} - 1\,303\,706 \times 1.015^{4n} + 1\,303\,706 \le 80\,000$$

$$303\,706 \times 1.015^{4n} \ge 1\,223\,706$$

$$1.015^{4n} \ge \frac{1223706}{303706}$$

$$4n \ln 1.015 \ge \ln \frac{1223706}{303706}$$

$$n \ge \frac{\ln \frac{1223706}{303706}}{4 \ln 1.015}$$

$$\ge 23.4000\ldots$$

∴ $80 000 can be withdrawn annually for 24 years. ✔

State Mean: **0.58/3**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by NESA.

Marking Feedback:

Students should:

- ☐ find the amount remaining in the account after the withdrawal at the end of the fourth and eighth quarters by explicitly showing the progression from A_1 to A_2
- ☐ present all working out with clear and sequential steps
- ☐ use patterns to generate an expression for A_n as a summation of terms
- ☐ use the correct value for the common ratio when finding the sum of a geometric progression formula

In better responses, students were able to:

☐ solve logarithmic equations

Areas for students to improve include:

☐ understanding the steps required to 'show' a result

☐ improving accuracy and skill in algebraic manipulation of logarithmic equations