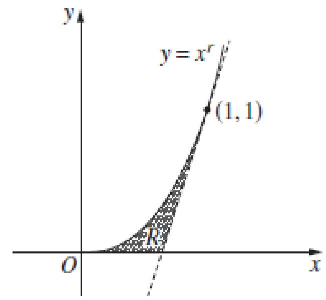
19 M **16 c** The diagram shows the region R, bounded by the curve $y = x^r$, where $r \geq 1$, the x-axis and the tangent to the curve at the point (1, 1).



(i) Show that the tangent to the curve at (1, 1) meets the x-axis at $\left(\frac{r-1}{r}, 0\right)$. **2**

(ii) Using the result of part (i), or otherwise, show that the area of the region R is $\frac{r-1}{2r(r+1)}$. **2**

(iii) Find the exact value of r for which the area of R is a maximum. **3**

(i) $y = x^r$

$$\frac{dy}{dx} = rx^{r-1}$$

Substitute $x = 1$:

$$\frac{dy}{dx}(1) = r(1)^{r-1} = r$$

Equation of tangent:

$$y - 1 = r(x - 1)$$

$$y = rx - r + 1 \quad ✔$$

Substitute $y = 0$:

$$0 = rx - r + 1$$

$$rx = r - 1$$

$$x = \frac{r-1}{r} \qquad \therefore R\left(\frac{r-1}{r}, 0\right) \quad ✔$$

(ii) $$\text{Area} = \int_0^1 x^r\,dx - \frac{1}{2} \times \left(1 - \frac{r-1}{r}\right) \times 1 \quad ✔$$

$$= \left[\frac{x^{r+1}}{r+1}\right]_0^1 - \frac{1}{2} \times \frac{r-r+1}{r} \times 1$$

$$= \left[\frac{1^{r+1}}{r+1} - \frac{0^{r+1}}{r+1}\right] - \frac{1}{2r}$$

$$= \frac{1}{r+1} - \frac{1}{2r}$$

$$= \frac{2r-(r+1)}{2r(r+1)}$$

$$= \frac{r-1}{2r(r+1)} \quad ✔$$

(iii) $$A = \frac{r-1}{2r(r+1)}$$

$$A = \frac{r-1}{2r^2+2r}$$

$$\frac{dA}{dr} = \frac{(2r^2+2r).1-(r-1)(4r+2)}{(2r^2+2r)^2} \quad ✔$$

$$= \frac{2r^2+2r-(4r^2-2r-2)}{(2r^2+2r)^2}$$

$$= \frac{-2r^2+4r+2}{(2r^2+2r)^2} = 0$$

$$r^2 - 2r - 1 = 0$$

$$r = \frac{2 \pm \sqrt{(-2)^2-4(1)(-1)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{8}}{2}$$

$$= \frac{2 \pm 2\sqrt{2}}{2}$$

$$= 1 + \sqrt{2} \quad ✔$$

(as $r > 0$)

Consider neighbourhood of $r = 1 + \sqrt{2}$:

r	2.4	$1+\sqrt{2}$	2.5
A'	> 0	0	< 0

$\therefore$ maximum area when $r = 1 + \sqrt{2}$. ✔

State Mean:
0.61/2
0.42/2
0.7/3

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Marking Feedback:

Students should:

☐ find the derivative of an expression with the treatment of the exponent of r

☐ realise that $1^{r+1} = 1$ to find the equation of the tangent

☐ use subtraction of areas to find the area of the given region

☐ use the quotient rule from the Reference Sheet, and remember to square the denominator

☐ justify the maximum value by using the simplest method

In better responses, students were able to:

☐ understand that the tangent meets the x-axis at $y = 0$

☐ clearly present all the steps in a 'show' question

☐ solve quadratic equations

☐ show numerical results when testing for concavity using the first derivative table

Areas for students to improve include:

☐ carefully examining the given diagram to select the most appropriate and simplest method

☐ considering the marks available for each part to gauge the required amount of working out

☐ using the answer provided in the question as a checking mechanism, rather than working backwards

☐ showing all algebraic manipulations in a 'show' question

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