20 MA **16** Sketch the graph of the curve $y = -x^3 + 3x^2 - 1$, labelling the stationary points and point of inflection. Do NOT determine the x intercepts of the curve. **4**

$y = -x^3 + 3x^2 - 1$

$\frac{dy}{dx} = -3x^2 + 6x = 0$ ✔

$-3x(x - 2) = 0$

$x = 0, 2$

Substitute into y:

$y(0) = -0^3 + 3(0)^2 - 1$

$= -1$

$y(2) = -(2)^3 + 3(2)^2 - 1$

$= 3$

∴ stationary points at (0, –1) and (2, 3). ✔

$\frac{d^2y}{dx^2} = -6x + 6$

$\frac{d^2y}{dx^2}(0) = -6(0) + 6 > 0$

∴ minimum (0, –1).

$\frac{d^2y}{dx^2}(2) = -6(2) + 6 < 0$

∴ maximum (2, 3).

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Also, $\frac{d^2y}{dx^2} = -6x + 6 = 0$

$6x = 6$

$x = 1$

Substitute into y:

$y(1) = -(1)^3 + 3(1)^2 - 1$

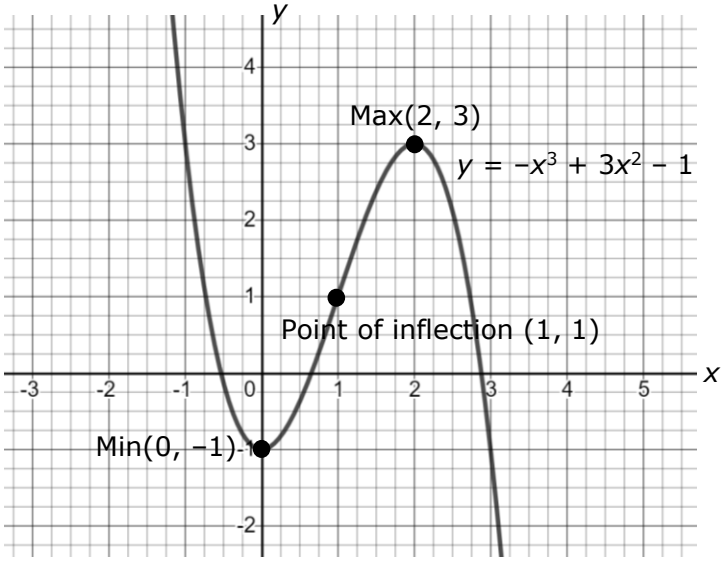
$= 1$

Possible point of inflection at (1, 1)

Consider neighbourhood of $x = 1$:

x	0	1	2
$\frac{d^2y}{dx^2}$	> 0	0	< 0

As sign change, then point of inflection at (1, 1). ✔



✔

HSC Marking Feedback

Question 16

Students should:

- avoid only using a table of values to sketch a curve
- ensure calculus is used to find stationary points and inflection points when sketching polynomial functions
- determine the nature of all stationary point they have found
- clearly label information derived for their curve on their graph
- set working out clearly and logically.

In better responses, students were able to:

- find the first and second derivatives
- find the stationary points by setting the first derivative equal to 0 and solving
- solve a quadratic equation
- find the point of inflection by setting the second derivative equal to 0 and solving
- find y-coordinates
- determine the nature of stationary points
- prove the concavity change for the point of inflection
- draw smooth curves and label important information on the drawing
- show all working.

Areas for students to improve include:

- practising simple differentiation, factorisation and substitution
- practising solving quadratic equations with a negative leading term
- drawing large diagrams that are fully labelled, with some thought about scale
- understanding how to determine the nature of stationary points
- understanding the difference between a stationary point and a point of inflection
- understanding the difference between a point of inflection and a horizontal point of inflection.

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by NESA.