**20 MA** **21** Hot tea is poured into a cup.

The temperature of tea can be modelled by $T = 25 + 70(1.5)^{-0.4t}$, where $T$ is the temperature of the tea, in degrees Celsius, $t$ minutes after it is poured.

(a) What is the temperature of the tea 4 minutes after it has been poured? **1**

(b) At what rate is the tea cooling 4 minutes after it has been poured? **2**

(c) How long after the tea is poured will it take for its temperature to reach 55 **3**

(a) $T = 25 + 70(1.5)^{-0.4t}$

Substitute $t = 4$:

$$T = 25 + 70(1.5)^{-0.4(4)}$$
$$= 61.58912515\ldots$$
$$= 61.6 \text{ (1 dec pl)}$$

$\therefore$ the temperature is 61.6°. ✔

(b) $\frac{dT}{dt} = 70 \times \ln 1.5 \times -0.4(1.5)^{-0.4t}$ ✔

$$= -28 \ln 1.5 \times (1.5)^{-0.4t}$$

Substitute $t = 4$:

$$\frac{dT}{dt}(4) = -28 \ln 1.5 \times (1.5)^{-0.4(4)}$$
$$= -5.934345433\ldots$$
$$= -5.9 \text{ (1 dec pl)}$$

$\therefore$ the temperature is dropping at 5.9°/min. ✔

(c) Substitute $T = 55$:

$$55 = 25 + 70(1.5)^{-0.4t}$$
$$70(1.5)^{-0.4t} = 30$$
$$1.5^{-0.4t} = \frac{3}{7}$$ ✔
$$\ln 1.5^{-0.4t} = \ln \frac{3}{7}$$
$$-0.4t \ln 1.5 = \ln \frac{3}{7}$$ ✔
$$-0.4t = \ln \frac{\ln \frac{3}{7}}{\ln 1.5}$$
$$t = -\frac{1}{0.4} \ln \frac{\ln \frac{3}{7}}{\ln 1.5}$$
$$t = 5.224234117\ldots$$
$$= 5 \text{ min } 13 \text{ sec (nearest sec)}$$

$\therefore$ it will take 5 minutes 13 seconds. ✔

State Mean:
**0.98/1**
**0.88/2**
**2.31/3**

## HSC Marking Feedback

### Question 21 (a)

**Students should:**

- use a calculator to find a numerical solution.

**In better responses, students were able to:**

- show substitution of $t = 4$ into the correct equation for temperature
- state the correct rounded answer for temperature.

**Areas for students to improve include:**

- refraining from writing a bald answer only
- correctly writing the answer from the calculator display.



### Question 21 (b)

**Students should:**

- differentiate exponential functions involving a base other than $e$

- understand the two-step solution requiring a substitution into the derivative
- differentiate a power function to find the instantaneous rate of change in a real-life situation
- state an appropriate conclusion regarding the instantaneous rate of change.

**In better responses, students were able to:**

- use the Reference Sheet to differentiate $70(1.5)^{-0.4t}$ correctly, showing all factors
- show substitution of $t = 4$ into $\frac{dT}{dt}$
- conclude with the correct answer for the rate.

**Areas for students to improve include:**

- understanding the difference between average rate and instantaneous rate
- identifying instantaneous rate as a derivative
- using the Reference Sheet to correctly differentiate exponentials with base other than $e$
- checking the calculator display when rounding.

## Question 21 (c)

**Students should:**

- develop, manipulate and solve an equation involving exponential functions
- recognise and use the inverse relationship between logarithms and exponential functions in a practical context
- use the formula $\frac{d}{dx}(a^x) = (\ln a)a^x$.

**In better responses, students were able to:**

- make $(1.5)^{-0.4t}$ the subject correctly
- take logarithms of both sides correctly
- apply logarithmic properties efficiently
- divide by a negative value correctly.

**Areas for students to improve include:**

- applying order of operations in the solution of an equation involving an exponential function
- demonstrating correct use of logarithmic laws
- refraining from using guess and check
- associating the concept of a rate to a derivative
- carefully checking values from one line of working to the next.