20 MA **23** A continuous random variable, X, has the following probability density functions.

$$f(x) = \begin{cases} \sin x & \text{for } 0 \le x \le k \\ 0 & \text{for all other values of k} \end{cases}$$

(a) Find the value of k. **2**

(b) Find $P(X \le 1)$. Give your answer correct to four decimal places. **2**

(a) $\int_0^k \sin x \, dx = 1$ ✔

$\left[-\cos x\right]_0^k = 1$

$-\cos k - (-\cos 0) = 1$

$-\cos k + 1 = 1$

$\cos k = 0$

$k = \frac{\pi}{2}$ ✔

(b) $P(X \le 1) = \int_0^1 \sin x \, dx$ ✔

$= \left[-\cos x\right]_0^1$

$= -\cos 1 - (-\cos 0)$

$= -\cos 1 + 1$

$= 0.459697694\ldots$

$= 0.4597$ (4 dec pl) ✔

State Mean:
1.20/2
0.87/2

HSC Marking Feedback

Question 23 (a)

Students should:

- understand the relationship between probability density functions and integration
- use the Reference Sheet to find the correct anti-derivative of $\sin x$
- recognise that the sum of all probabilities of a probability density function is equal to 1, ie the definite integral is equal to 1
- know how to find k using the probability density function.

In better responses, students were able to:

- display an understanding of the definition of a probability density function
- integrate correctly
- substitute bounds correctly
- solve the trigonometric equation, and recognise the need for radians in the context of calculus.

Areas for students to improve include:

- using the Reference Sheet for probability density functions
- solving trigonometric equations
- using the Reference Sheet to correctly express the anti-derivative of $\sin x$ as $-\cos x$ as opposed to $\cos x$
- knowing that the area under a probability density function is equal to 1
- understanding the need to use radians in calculus.

Question 23 (b)

Students should:

- understand the link between continuous random variables and probability
- use calculus to find probabilities
- show all steps involved in evaluating a definite integral
- understand the need for radians in calculus.

In better responses, students were able to:

- write down the correct definite integral
- integrate and substitute bounds correctly
- evaluate correctly, using radians
- round answer to 4 decimals places correctly as stated in the question.

Areas for students to improve include:

- finding the correct definite integral
- showing the ability to integrate correctly and apply correct limits
- giving a solution correctly in radians
- understanding the meaning of $P(X \leq 1)$ as equating to evaluating the integral of the probability density function between 0 and 1.