20 MA **26** Tina inherits \$60 000 and invests it in an account earning interest at a rate of 0.5% per month. Each month, immediately after the interest has been paid, Tina withdraws \$800.

The amount in the account immediately after nth withdrawal can be determined using the recurrence relation $A_n = A_{n-1}(1.005) - 800$, where $n = 1, 2, 3, \ldots$ and $A_0 = 60\,000$.

(a) Use the recurrence relation to find the amount of money in the account immediately after the third withdrawal. **2**

(b) Calculate the amount of interest earned in the first three months. **2**

(c) Calculate the amount of money in the account immediately after the 94th withdrawal. **3**

(a) $A_1 = A_0(1.005) - 800$

$= 60\,000(1.005) - 800$

$= 59\,500$ ✔

$A_2 = A_1(1.005) - 800$

$= 59\,500(1.005) - 800$

$= 58\,997.50$

$A_3 = A_2(1.005) - 800$

$= 58\,997.50(1.005) - 800$

$= 58\,492.49$ (2 dec pl) $\therefore$ the amount of money in the account was \$58 492.49. ✔

(b) Interest $= 60000 \times 0.005 + 59500 \times 0.005 + 58977.50 \times 0.005$ ✔

$= 892.4875$

$= 892.49$ (2 dec. pl.) $\therefore$ the interest earned was \$892.49. ✔

State Mean:
1.69/2
1.24/2
1.59/3

(c) $A_1 = 60\,000(1.005) - 800$

$A_2 = [60\,000(1.005) - 800] \times 1.005 - 800$

$= 60\,000(1.005)^2 - 800(1 + 1.005)$

Hence $A_{94} = 60\,000(1.005)^{94} - 800(1 + 1.005 + 1.005^2 + \ldots + 1.005^{93})$ ✔ ✔

$$= 60\,000(1.005)^{94} - 800 \times \frac{1(1.005^{94} - 1)}{1.005 - 1}$$

(Using a sum of a geometric series with $a = 1$, $r = 1.005$, $n = 94$)

$= 187.8459979\ldots$

$= 187.85$ $\therefore$ there will be \$187.85 in the account. ✔

HSC Marking Feedback

Question 26 (a)

Students should:

- understand the mathematics of a recurrence relationship
- use a calculator efficiently, including correct rounding

- substitute the values correctly to evaluate the final answer
- understand subscript notation.

In better responses, students were able to:

- calculate the value of the first and the second amount
- use the recurrence relationship correctly to calculate A_3
- use an algebraic approach to the recurrence relationship and evaluate with a calculator in the final step only.

Areas for students to improve include:

- practising using a recurrence relationship
- showing clearly set out working.

Question 26 (b)

Students should:

- remember that parts of questions are often related
- use the solution of part (a) to calculate the solution to this part
- calculate each successive year's interest and adding them.

In better responses, students were able to:

- recognise that the interest was the difference between the repayments and the reduction of the principal value
- calculate interest for each successive month
- understand that this section was not a simple interest or compound interest calculation
- obtain the balance reduced by subtracting the initial amount from their amount of money in the account immediately after the third withdrawal.

Areas for students to improve include:

- avoiding overcomplicating their thinking
- understanding the different ways of calculating interest.

Question 26 (c)

Students should:

- set up the geometric series using the first 3 terms of the series: A_1, A_2 and A_3
- write the 94th term of the series
- use the sum of a geometric series with the correct number of terms
- avoid using a calculator 94 times to find the answer.

In better responses, students were able to:

- show their working clearly to demonstrate the formation of the geometric series
- understand that the series has 94 terms to sum
- remove the common factor of 800 correctly
- use the sum of a geometric series to calculate the solution.

Areas for students to improve include:

- practising using a calculator for the sum of a geometric series
- showing clear and concise working out.