20 MA **28** In a particular country, the hourly rate of pay for adults who work is normally distributed with a mean of \$25 and a standard deviation of \$5.

(a) Two adults who both work are chosen at random. **3**

Find the probability that at least one of them earns between \$15 and \$30 per hour.

(b) The number of adults who work is equal to three times the number of adults who do not work. **2**

One adult is chosen at random.

Find the probability that the chosen adult works and earns more than \$25 per hour.

(a) $\mu = 25$ and $\sigma = 5$ and $z = \dfrac{x-\mu}{\sigma}$:

When $x = 15$, $z = \dfrac{15-25}{5}$

$= -2$

When $x = 30$, $z = \dfrac{30-25}{5}$

$= 1$

P(between \$15 and \$30)

$= P(-2 < z < 1)$

$= P(z < 1) - P(z < -2)$

$= 0.84 - 0.025$

$= 0.815$ ✔

$\therefore$ P(does not earn between \$15 and \$30)

$= 1 - 0.815$

$= 0.185$ ✔

P(at least one earns between \$15 and \$30)

$= 1 - P$(none earns between \$15 and \$30)

$= 1 - 0.185^2$

$= 0.965775$

$= 0.966$ (3 dec pl) ✔

(b) P(adult works) $= \dfrac{3}{4}$, ✔

P(earns more than \$25) $= \dfrac{1}{2}$.

P(adult works and earns more than \$25/h)

$= \dfrac{3}{4} \times \dfrac{1}{2}$

$= \dfrac{3}{8}$ ✔

State Mean:
1.21/3
1.15/2

HSC Marking Feedback

Question 28 (a)

Students should:

- use the empirical results of a normal distribution
- link probability with a normal distribution
- find the probability of a given event using z-scores.

In better responses, students were able to:

- find the required z-scores
- calculate the probability of one adult within the given range of values from the normal distribution
- recognise the problem as a two-stage event

- use a tree diagram to illustrate the sample space for a two-stage event
- recognise that 'at least one' means one or more
- recognise 'at least' as a complementary event
- understand that the area above the mean in a normal distribution is $\frac{1}{2}$.

Areas for students to improve include:

- using a diagram to represent the normal distribution
- referring to the Reference Sheet for percentages between z-scores
- understanding that the normal distribution is a bell-shaped curve
- using a tree diagram for multi-stage events
- showing all steps in calculation of the required probability.

Question 28 (b)

Students should:

- use complementary events to calculate probability
- recognise that the sum of the probabilities is equal to 1
- find the probability of two simultaneous events.

In better responses, students were able to:

- interpret a ratio as a probability
- identify the mean as having a z-score of 0 and a probability of $\frac{1}{2}$
- clearly state the probabilities as given in the question
- recognise that $P(A \text{ and } B) = P(A) \times P(B)$.

Areas for students to improve include:

- recognising the difference between two-stage probability and conditional probability
- defining the independent probabilities
- writing the % sign if the value represents a percentage
- understanding the role of the mean in a probability distribution
- identifying the conjunction 'and' in a probability context to imply multiplication.