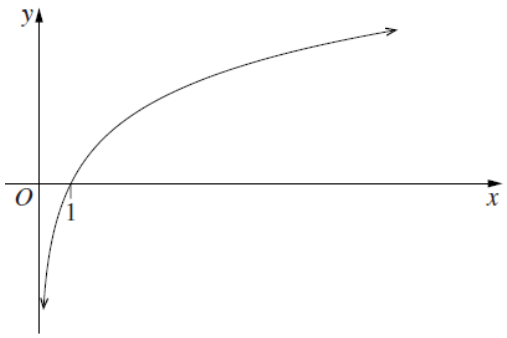
20 MA **29** The diagram shows the graph of $y = c\ln x$, $c > 0$.



(a) Show that the equation of the tangent to $y = c\ln x$, at $x = p$, where $p > 0$ is

$$y = \frac{c}{p}x - c + c\ln p.$$ **2**

(b) Find the value of c such that the tangent from part (a) has a gradient of 1 and passes through the origin. **2**

(a) $y = c\ln x$

$$\frac{dy}{dx} = \frac{c}{x}$$

$$\frac{dy}{dx}(p) = \frac{c}{p}$$ ✔

Also, substitute $x = p$ in y:

$y(p) = c\ln p$

Using $(p, c\ln p)$ and gradient $= \frac{c}{p}$:

$$y - c\ln p = \frac{c}{p}(x - p)$$

$$y - c\ln p = \frac{c}{p}x - c$$

$$y = \frac{c}{p}x - c + c\ln p$$ ✔

(b) $y = \frac{c}{p}x - c + c\ln p$

As passes through origin, substitute $x = 0$, $y = 0$:

$$0 = \frac{c}{p}(0) - c + c\ln p$$ ✔

$$c = c\ln p$$

$$1 = \ln p$$

$$p = e$$

As $\frac{dy}{dx}(p) = \frac{c}{p}$, then $\frac{dy}{dx}(e) = \frac{c}{e} = 1$

$$\therefore\ c = e$$ ✔

State Mean:
1.06/2
0.79/2

HSC Marking Feedback

Question 29 (a)

Students should:

- differentiate a logarithmic function
- determine the gradient at a given point
- calculate the y-coordinate at a given point
- derive the equation of a tangent using the point-gradient formula.

In better responses, students were able to:

- find the correct derivative of the given log function
- calculate the gradient of the tangent at $x = p$
- find the y-coordinate at $x = p$
- use point-gradient formula to find the equation of the tangent.

Areas for students to improve include:

- identifying c as a constant when taking the derivative of the logarithmic function

- differentiating logarithmic functions
- stating the derivative in terms of x
- noting that a point coincides with a line if it satisfies the equation of that line
- substituting the x-coordinate into the derivative to find the gradient before substituting into a formula
- showing the substitution of values into the formulae.

Question 29 (b)

Students should:

- substitute a coordinate into the equation of a tangent to find an unknown value
- solve a logarithmic equation.

In better responses, students were able to:

- set the gradient from part (a) equal to 1
- write the relationship between c and p
- substitute $(0, 0)$ into the equation of the tangent
- solve an equation involving logarithms
- use simultaneous equations to solve the equation for c.

Areas for students to improve include:

- identifying the gradient in the equation of a straight line
- showing correct substitution of a point into a linear equation
- solving equations containing logarithms
- clearly writing the solution for the required variable.