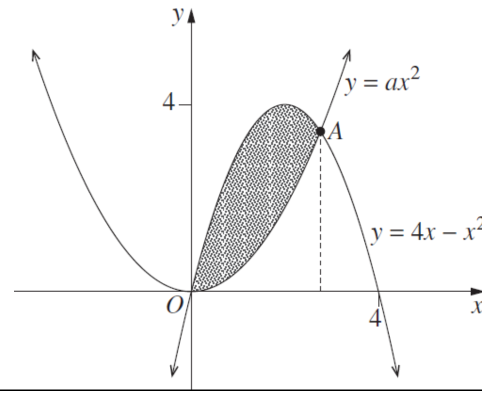
20 MA **30** The diagram shows two parabolas $y = 4x - x^2$ and $y = ax^2$, where $a > 0$.

The two parabolas intersect at the origin, O, and at A.



(a) Show that the x-coordinate of A is $\frac{4}{a+1}$. **2**

(b) Find the value of a such that the shaded area is $\frac{16}{3}$. **4**

(a) $ax^2 = 4x - x^2$

$ax^2 + x^2 - 4x = 0$ ✔

$x^2(a + 1) - 4x = 0$

$x[x(a + 1) - 4] = 0$

$x = 0 \quad x(a + 1) = 4$

$$x = \frac{4}{a+1}$$

$\therefore$ A has x-coordinate of $\frac{4}{a+1}$. ✔

(b) Area $= \int_0^{\frac{4}{a+1}} \left(4x - x^2 - ax^2\right) dx = \frac{16}{3}$ ✔

$$\int_0^{\frac{4}{a+1}} \left(4x - (1+a)x^2\right) dx = \frac{16}{3}$$

$$\left[2x^2 - (a+1)\frac{x^3}{3}\right]_0^{\frac{4}{a+1}} = \frac{16}{3}$$ ✔

$$2\left(\frac{4}{a+1}\right)^2 - (a+1)\frac{\left(\frac{4}{a+1}\right)^3}{3} - 0 = \frac{16}{3}$$ ✔

$$\frac{32}{(a+1)^2} - \frac{64(a+1)}{3(a+1)^3} = \frac{16}{3}$$

$$\frac{32}{(a+1)^2} - \frac{64}{3(a+1)^2} = \frac{16}{3}$$

$96 - 64 = 16(a + 1)^2$

(by multiplying by $3(a + 1)^2$)

$16(a + 1)^2 = 32$

$(a + 1)^2 = 2$

$a + 1 = \pm\sqrt{2}$

$a = -1 + \sqrt{2}$ ✔

(as $a > 0$)

State Mean:
1.36/2
1.93/4

HSC Marking Feedback

Question 30 (a)

Students should:

- show their working clearly and coherently moving from one step to the next
- include many steps in their proofs
- read the question carefully to understand what result they are required to reach.

In better responses, students were able to:

- solve the quadratic equations simultaneously
- factorise carefully
- use brackets correctly
- clearly write the steps involved to 'show' the given result.

- use brackets correctly
- clearly write the steps involved to 'show' the given result.

Areas for students to improve include:

- practising 'show' questions
- knowing the different methods of solving quadratic equations.

Question 30 (b)

Students should:

- know and apply each step involved in finding a definite integral
- substitute both the limits into a primitive in the correct order
- solve carefully for a, being mindful of careless algebraic errors
- check their two solutions for the value of a to ensure their final answer is valid
- factorise like terms before finding a primitive.

In better responses, students were able to:

- understand the link to part (a) in writing the correct definite integral
- understand how to find the area between two curves
- obtain a correct integral containing correct quadratic expression and integrate correctly
- substitute the correct limits, including 0
- recognise the link made through factorising out x^2 before the integration step, or factorising out x^3 before solving for a
- recognise the fact that the value of a must be positive in their final solution.

Areas for students to improve include:

- using general algebra skills in factorising and solving equations
- knowing the processes involved in evaluating a definite integral.

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by NESA.