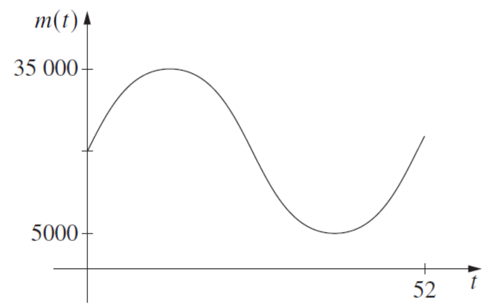
20 MA **31** The population of mice on an isolated island can be modelled by the function $m = a\sin\left(\frac{\pi}{26}t\right) + b$, where t is the time in weeks and $0 \le t \le 52$. The population of mice reaches a maximum of 35 000 when $t = 13$ and a minimum of 5000 when $t = 39$.



The graph of $m(t)$ is shown.

(a) What are the values of a and b? **2**

(b) On the same island, the population of cats can be modelled by the function **3**

$$c(t) = -80\cos\left(\frac{\pi}{26}(t-10)\right) + 120.$$

Consider the graph of $m(t)$ and the graph of $g(t)$.

Find the values of t, $0 \le t \le 52$, for which both populations are increasing.

(c) Find the rate of change of the mice population when the cat population reaches a maximum. **2**

(a) Average of max and min

$$= \frac{35000 + 5000}{2}$$

$$= 20\,000.$$

This means $b = 20\,000$.

Hence, $a = 35\,000 - 20\,000$

$= 15\,000$

$\therefore\ a = 15\,000,\ b = 20\,000$

(b) From the graph, $m(t)$ is increasing when

$0 < t < 13$ and $39 < t \le 52$.

As $f(x) = \cos x$ is decreasing from $0 < t < \pi$, etc, then $f(x) = -\cos x$ is increasing from $0 < t < \pi$, etc.

This means $c(t)$ is increasing when

$$0 < \left(\frac{\pi}{26}(t-10)\right) < \pi$$

$$0 < t - 10 < \frac{\pi}{\frac{\pi}{26}}$$

$$0 < t - 10 < 26$$

$$10 < t < 36$$

Hence, both pops are increasing when $10 < t < 13$.

(c) $c(t)$ reaches a maximum when $t = 36$.

$$m(t) = 15\,000\sin\left(\frac{\pi}{26}t\right) + 20\,000$$

$$m'(t) = \frac{15\,000\pi}{26}\cos\left(\frac{\pi}{26}t\right)$$

$$m'(36) = \frac{15\,000\pi}{26}\cos\frac{36\pi}{26}$$

$$= -642.7062162\ldots$$

$$= -643 \text{ (nearest whole)}$$

$\therefore$ the mice population is decreasing at the rate of 643 per week.

State Mean:
1.33/2
0.90/3
0.53/2

HSC Marking Feedback

Question 31 (a)

Students should:

- understand that the trigonometric graph is written with radians as the unit of measurement, not degrees
- understand what the terms a and b represent in a trigonometric function
- know the correct position of amplitude in a trigonometric equation.

In better responses, students were able to:

- understand the terms amplitude and translation in the process, while linking to the sine curve
- understood the geometrical significance of a and b and read their values from the graphs
- solve simultaneous equations effectively.

Areas for students to improve include:

- taking care to avoid careless errors when finding the median point on the graph at the y-intercept
- understanding amplitude and the translation of trigonometric functions
- reading information from a graph.

Question 31 (b)

Students should:

- know how to work with trigonometric functions in term of π
- know how to find the derivative of trigonometric functions.

In better responses, students were able to:

- use the graphs to find where both populations were increasing, making a clear link to what they understood from Question 30 (a)
- find both $c'(t)$ and $m'(t)$ and interpret their application to the term 'increasing'
- apply an understanding of where the stationary points on a trigonometric graph exist (for sine and cosine curves), and link these to the correct notation for displaying an inequality
- use inequalities correctly to express the solution, understanding the difference between 'less than' and 'less than or equal to'.

Areas for students to improve include:

- understanding the trigonometric graphs and how to draw and interpret them
- understanding the rules for finding the derivative of trigonometric functions
- understanding the difference between differentiation and integration and their uses in solving problems.

Question 31 (c)

Students should:

- show numerical results when completing the test for the maximum or minimum
- appreciate rate of change as the first derivative of $m(t)$
- accurately substitute and evaluate expressions
- show their substitution step.

In better responses, students were able to:

- find the correct derivative of $m'(t)$ and substitute accordingly
- link their understanding of what they were being asked to do in parts (a) and (b)
- find $t = 36$ as the maximum value and the substitute into $m'(t)$
- reach a correct maximum value through calculation relating to a derivative, or through sketching the correct trigonometric graph
- draw a graph to show the population of cats and then found correct inequalities.

Areas for students to improve include:

- understanding the rules for differentiation of complex trigonometric functions
- drawing clear graphs.

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by NESA.