MA 18 The diagram shows a continuous function SP $\quad \begin{array}{cc}\text { Band } \\ 2-5\end{array} \quad y=f(x)$ defined in the domain $[-2,10]$. The function consists of a quarter of a circle centred at $(0,0)$ with radius 2 , a straight line segment and a logarithmic function $f(x)=\ln (x-2)$ in the domain $[3,10]$.
(a) Find the exact area bounded by the function $y=f(x)$ and the $x$-axis in the domain $[-2,3]$.
(b) Hence, find the exact value of


$$
\int_{3}^{10} \ln (x-2) d x, \text { given that } \int_{-2}^{10} f(x) d x=8 \ln 8-10-\pi
$$

(a) Area $=\int_{-2}^{3} \ln (x-2) d x$

$$
=\text { Area of quadrant }+ \text { area of triangle }
$$

$$
=\frac{1}{4} \times \pi \times 2^{2}+\frac{1}{2} \times 2 \times 3
$$

$$
=\pi+3 \quad \therefore \text { the area is }(\pi+3) \text { units }^{2}
$$

(b)

$$
\begin{aligned}
\int_{-2}^{10} f(x) d x & =\int_{-2}^{3} \ln (x-2) d x+\int_{3}^{10} \ln (x-2) d x \\
8 \ln 8-10-\pi & =-(\pi+3)+\int_{3}^{10} \ln (x-2) d x \\
\int_{3}^{10} \ln (x-2) d x & =8 \ln 8-7
\end{aligned}
$$

* These solutions have been provided by projectmaths and are not supplied or endorsed by NESA.

