MA SP **36** Band 2-6

An island initially has 16 100 trees. The number of trees increases by 1% per annum. The people on the island cut down 1161 trees at the end of each year.

(a) Show that after the first year there are 15 100 trees. **1**

(b) Show that at the end of 2 years the number of trees remaining is given by the expression $T_2 = 16\,100 \times (1.01)^2 - 1161(1 + 1.01)$. **2**

(c) Show that at the end of n years the number of trees remaining is given by the expression $T_n = 116\,100 - 100\,000 \times (1.01)^n$. **2**

(d) For how many years will the people on the island be able to cut down 1161 trees annually? **1**

(a) $T_1 = 16\,100 \times 1.01 - 1161$

$= 15\,100$

(b) $T_2 = T_1 \times 1.01 - 1161$

$= (16\,100 \times 1.01 - 1161) \times 1.01 - 1161$

$= 16\,100 \times (1.01)^2 - 1161 \times 1.01 - 1161$

$= 16\,100 \times (1.01)^2 - 1161(1 + 1.01)$

(c) Using this recursive rule,

$T_3 = 16\,100 \times (1.01)^3 - 1161(1 + 1.01 + 1.01^2)$

Similarly,

$T_n = 16\,100 \times (1.01)^n - 1161(1 + 1.01 + 1.01^2 + \dots + 1.01^{n-1})$

Using $a = 1$, $r = 1.01$, $n = n$, $S_n = \dfrac{a(r^n - 1)}{r - 1}$:

$T_n = 16\,100 \times (1.01)^n - 1161\left[\dfrac{1(1.01^n - 1)}{1.01 - 1}\right]$

$= 16\,100 \times (1.01)^n - 116\,100(1.01^n - 1)$

$= 16\,100 \times (1.01)^n - 116\,100 \times (1.01)^n + 116\,100$

$= 116\,100 - 100\,000 \times (1.01)^n$

(d) $116\,100 - 100\,000 \times (1.01)^n = 0$

$100\,000 \times (1.01)^n = 116\,100$

$1.01^n = 1.161$

$\ln 1.01^n = \ln 1.161$

$n \ln 1.01 = \ln 1.161$

$n = \dfrac{\ln 1.161}{\ln 1.01}$

$= 15.00268734\dots$ … $\therefore$ 15 years.

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by NESA.