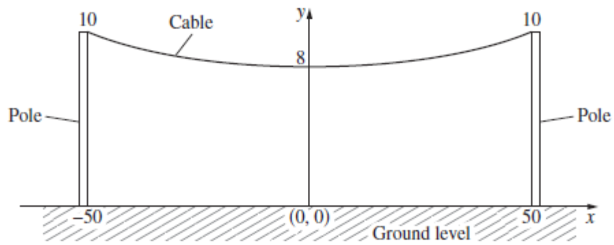
MA SP **38** Band 3-6

A cable is freely suspended between two 10 m poles, as shown.
The poles are 100 m apart and the minimum height of the cable is 8 metres.
The height of the cable is given as $y = c(e^{kx} + e^{-kx})$, where c and k are positive constants.



(a) Show that the value of c is 4. **1**

(b) Use the result in part (a) to show that one value of k is $\dfrac{\ln 2}{50}$. **4**

(c) Hence find the area between the poles, the cable and the ground. **3**

(a) Substitute $x = 0$, $y = 8$:

$$y = c(e^{kx} + e^{-kx})$$
$$8 = c(e^{k(0)} + e^{-k(0)})$$
$$8 = c(1 + 1)$$
$$2c = 8$$
$$c = 4$$

(b) Substitute $x = 50$, $y = 10$:

$$y = 4(e^{kx} + e^{-kx})$$
$$10 = 4(e^{k(50)} + e^{-k(50)})$$
$$e^{50k} + e^{-50k} = 2.5$$

Let $m = e^{50k}$:

$$m + m^{-1} = 2.5$$

Multiplying through by m:

$$m^2 + 1 = 2.5m$$
$$2m^2 - 5m + 2 = 0$$
$$(2m - 1)(m - 2) = 0$$
$$m = \frac{1}{2}, 2$$

$$e^{50k} = \frac{1}{2} \qquad e^{50k} = 2$$
$$50k = \ln\frac{1}{2} \qquad 50k = \ln 2$$
$$k = -\frac{\ln 2}{50} \qquad k = \frac{\ln 2}{50}$$

Hence $k = \dfrac{\ln 2}{50}$ is a solution.

(c) $y = 4(e^{kx} + e^{-kx})$

$$\text{Area} = 2\int_0^{50} 4(e^{kx} + e^{-kx})\,dx$$
$$= 8\int_0^{50} \left(e^{kx} + e^{-kx}\right)dx$$
$$= 8\left[\frac{1}{k}e^{kx} - \frac{1}{k}e^{-kx}\right]_0^{50}$$
$$= \frac{8}{k}\left[e^{kx} - e^{-kx}\right]_0^{50}$$
$$= \frac{8}{k}[e^{50k} - e^{-50k} - (e^0 - e^0)]$$
$$= \frac{8}{k}[e^{50k} - e^{-50k}]$$

Now, substituting $k = \dfrac{\ln 2}{50}$:

$$\text{Area} = \frac{400}{\ln 2}[2 - \frac{1}{2}]$$
$$= 865.6170245\ldots$$
$$= 866 \text{ (nearest whole)}$$

$\therefore$ area is 866 units^2