MA 38 A cable is freely suspended between
SP $\quad \begin{aligned} & \text { Band } \\ & 3-6\end{aligned}$ two 10 m poles, as shown.
The poles are 100 m apart and the minimum height of the cable is 8 metres.
The height of the cable is given as $y=c\left(e^{k x}+e^{-k x}\right)$, where $c$ and $k$ are positive constants.

(a) Show that the value of $c$ is 4 .
(b) Use the result in part (a) to show that one value of $k$ is $\frac{\ln 2}{50}$.
(c) Hence find the area between the poles, the cable and the ground.
(a) Substitute $x=0, y=8$ :

$$
\begin{aligned}
y & =c\left(e^{k x}+e^{-k x}\right) \\
8 & =c\left(e^{k(0)}+e^{-k(0)}\right) \\
8 & =c(1+1) \\
2 c & =8 \\
c & =4
\end{aligned}
$$

(b) Substitute $x=50, y=10$ :

$$
\begin{aligned}
y & =4\left(e^{k x}+e^{-k x}\right) \\
10 & =4\left(e^{k(50)}+e^{-k(50)}\right) \\
e^{50 k}+e^{-50 k} & =2.5
\end{aligned}
$$

Let $m=e^{50 k}$ :

$$
m+m^{-1}=2.5
$$

Multiplying through by m :

$$
\begin{aligned}
& m^{2}+1=2.5 m \\
& 2 m^{2}-5 m+2=0 \\
&(2 m-1)(m-2)=0 \\
& m= \frac{1}{2}, 2 \\
& e^{50 k}=\frac{1}{2} \quad e^{50 k}=2 \\
& 50 k=\ln \frac{1}{2} \quad 50 k=\ln 2 \\
& k=-\frac{\ln 2}{50} \quad k=\frac{\ln 2}{50} \\
& \text { Hence } k=\frac{\ln 2}{50} \text { is a solution. }
\end{aligned}
$$

(c) $y=4\left(e^{k x}+e^{-k x}\right)$

Area $=2 \int_{0}^{50} 4\left(e^{k x}+e^{-k x}\right) d x$

$$
=8 \int_{0}^{50}\left(e^{k x}+e^{-k x}\right) d x
$$

$$
=8\left[\frac{1}{k} e^{k x}-\frac{1}{k} e^{-k x}\right]_{0}^{50}
$$

$$
=\frac{8}{k}\left[e^{k x}-e^{-k x}\right]_{0}^{50}
$$

$$
=\frac{8}{k}\left[e^{50 k}-e^{-50 k}-\left(e^{0}-e^{0}\right)\right]
$$

$$
=\frac{8}{k}\left[e^{50 k}-e^{-50 k}\right]
$$

Now, substituting $k=\frac{\ln 2}{50}$ :

$$
\begin{aligned}
\text { Area } & =\frac{400}{\ln 2}\left[2-\frac{1}{2}\right] \\
& =865.6170245 \ldots \\
& =866 \text { (nearest whole) }
\end{aligned}
$$

$\therefore$ area is 866 units $^{2}$

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