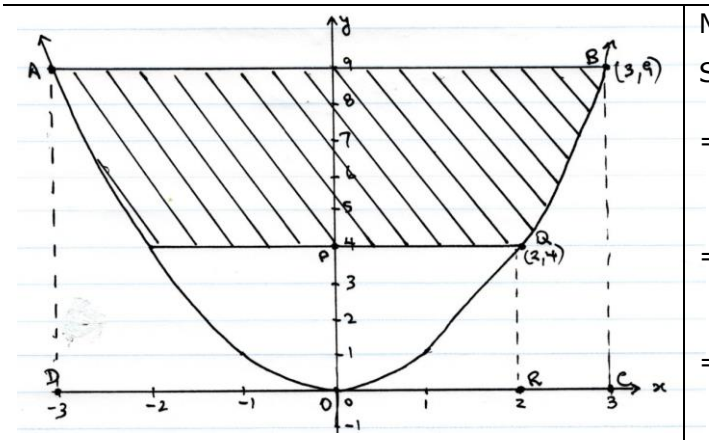
TG ADI **9** Sketch the region bounded by the curve $y = x^2$ and the lines $y = 4$, $y = 9$. Evaluate the area of this region.



Method 1:

As $y = x^2$, then $x = \sqrt{y}$ and find the area between the curve and the y-axis:

$$\text{Area} = 2\int_4^9 y^{\frac{1}{2}}\,dy$$

$$= 2\left[\frac{2y^{\frac{3}{2}}}{3}\right]_4^9$$

$$= \frac{4}{3}\left[y^{\frac{3}{2}}\right]_4^9$$

$$= \frac{4}{3}\left[9^{\frac{3}{2}} - 4^{\frac{3}{2}}\right]$$

$$= \frac{4}{3}(27 - 8)$$

$$= 25\frac{1}{3} \qquad \therefore \text{ area is } 25\frac{1}{3} \text{ units}^2.$$

Method 2:

Shaded area

$$= \text{Area } ABCD - (2 \times \text{area } PQRO + 2 \times \int_2^3 x^2\,dx)$$

$$= 9 \times 6 - 2(4 \times 2 + \left[\frac{x^3}{3}\right]_2^3)$$

$$= 54 - 2(8 + \left[\frac{3^3}{3} - \frac{2^3}{3}\right])$$

$$= 54 - 2(8 + 6\frac{1}{3})$$

$$= 25\frac{1}{3} \qquad \therefore \text{ area is } 25\frac{1}{3} \text{ units}^2.$$