TG 16 M **1 14 b** A gardener develops an eco-friendly spray that will kill harmful insects on fruit trees without contaminating the fruit. A trial is to be conducted with 100 000 insects. The gardener expects the spray to kill 35% of the insects each day and exactly 5000 new insects will be produced each day.

The number of insects expected at the end of the nth day of the trial is A_n.

(i) Show that $A_2 = 0.65(0.65 \times 100\,000 + 5000) + 5000$. **2**

(ii) Show that $A_n = 0.65^n \times 100\,000 + 5000\dfrac{(1-0.65^n)}{0.35}$. **1**

(iii) Find the expected insect population at the end of the fourteenth day, correct to the nearest 100. **1**

(i) As 35% of insects die, then 65% of insects survive.

$\therefore A_1 = 0.65 \times 100\,000 + 5000$

$A_2 = 0.65 \times A_1 + 5000$

$= 0.65(0.65 \times 100\,000 + 5000) + 5000$

State Mean: **1.65**

(ii) $A_2 = 0.65^2 \times 100\,000 + 0.65 \times 5000 + 5000$

$= 0.65^2 \times 100\,000 + 5000(1 + 0.65)$

$\therefore A_n = 0.65^n \times 100\,000 + 5000(1 + 0.65 + 0.65^2 + \ldots + 0.65^{n-1})$

Using a geometric sum with $a = 1$, $r = 0.65$,

$n = n$, $S_n = \dfrac{a(1-r^n)}{1-r}$:

$A_n = 0.65^n \times 100\,000 + 5000\left[\dfrac{1(1-0.65^n)}{1-0.65}\right]$

$= 0.65^n \times 100\,000 + 5000\dfrac{(1-0.65^n)}{0.35}$

State Mean: **0.60**

(iii) Let $n = 14$:

$A_{14} = 0.65^{14} \times 100\,000 + 5000\dfrac{(1-0.65^{14})}{0.35}$

$= 14\,491.70147\ldots$

$= 14\,500$ (nearest hundred)

$\therefore$ the expected population of 14 500.

State Mean: **0.84**

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by NESA

BOSTES: Notes from the Marking Centre

Most showed a good understanding of how to develop the series and identify it as a GP.

(i) The majority showed an understanding of the nature of the question and the link between A1 and A2. Common problems were:

- using 1.65 or 0.35 instead of 0.65
- finding A_1 only
- deriving an incorrect expression for A_1 leading to an incorrect expression for A_2
- incorrect use or omission of brackets when writing the expressions for A_1 and A_2 or inconsistently omitting zeros in 100 000 or 5000.

(ii) Most showed the progression from A2 to A3 and onto An, displaying the geometric series with three consecutive terms and the last term, and correctly substituting into the GP sum formula. Common problems were:

- attempting to identify a GP with only 2 terms
- attempting to work backwards from the required result
- omitting the first or last term of the GP.

(iii) This question was completed well by most candidates. Common problems were:

- calculator errors
- misinterpreting the statement 'correct to the nearest 100' to mean $n = 100$, hence incorrectly substituting 100 into the formula
- using $n = 13$ or $n = 15$.