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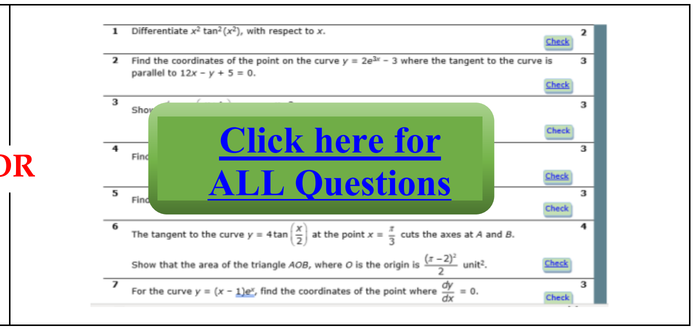
MathsFit

Mathematics Advanced

Yr 12 Calculus

10. Area Under Curve A

OR



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- choose individual question: **Questions** 1 2 3 ...
- check solution: Check
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Yr 12 Calculus: Area Under Curve A

1 The following table lists the values of a function $y = f(x)$ for 3 values of x. **1**

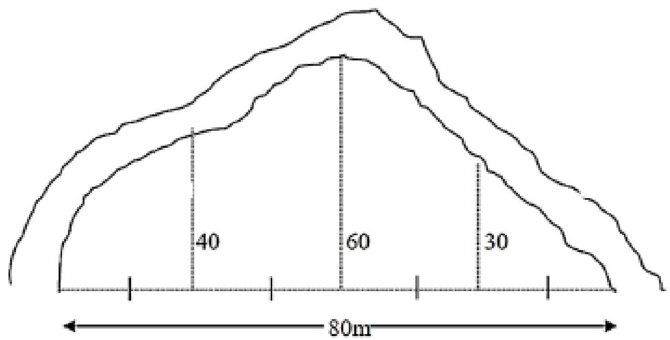
x	1.2	1.4	1.6
y	3	3.8	4.8

By using the trapezoidal rule and the table of values, the best estimation of $\int_{1.2}^{1.6} f(x)dx$ is:

A. 3.08 B. 1.54 C. 0.77 D. 7.7

Check

2 A paddock is bounded by a fence and a river as illustrated below: **1**

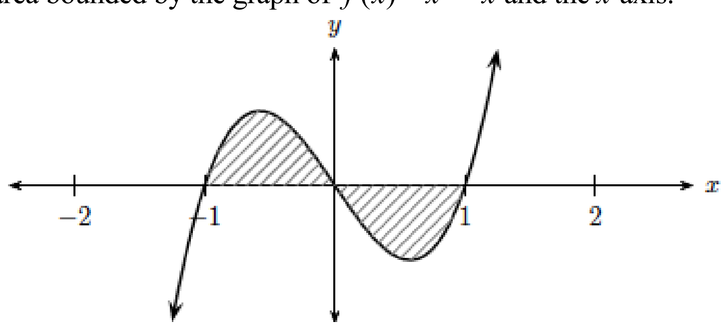


Four applications of the trapezoidal rule were used to determine the area of the paddock.
What is the approximate area of the paddock?

A. 1300 m^2 B. 1900 m^2 C. 2600 m^2 D. 5200 m^2

Check

3 The diagram shows the area bounded by the graph of $f(x) = x^3 - x$ and the x-axis. **1**

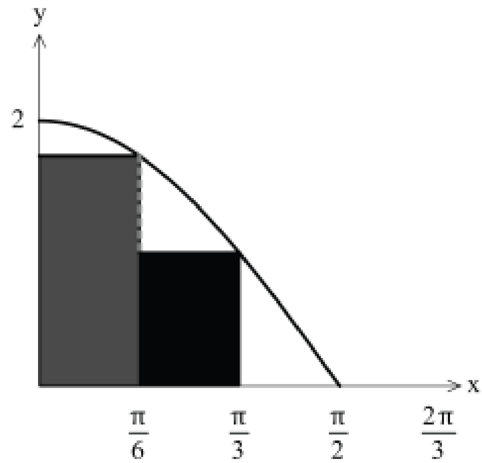


Given that $f(x)$ is an odd function, which of the following correctly gives the shaded area?

A. $2\int_{-1}^{0}\left(x^3 - x\right)dx$ B. $1\int_{0}^{1}\left(x^3 - x\right)dx$ C. $\int_{-1}^{1}\left(x^3 - x\right)dx$ D. $2\int_{-1}^{1}\left(x^3 - x\right)dx$

Check

4 The area under the curve $y = 2\cos x$, as shown below, is approximated by two rectangles. **1**



The value of the approximation is:

A. 1 B. $\dfrac{\pi\left(\sqrt{3}+1\right)}{6}$ C. $\sqrt{3}+1$ D. $2\left(\dfrac{\pi}{6}+\dfrac{\pi}{3}\right)$

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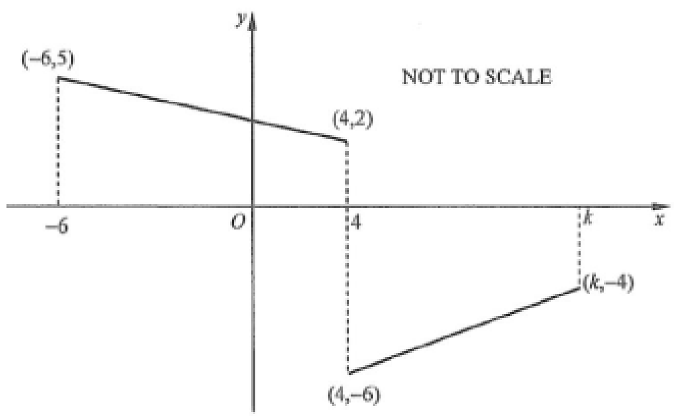
5 **1**

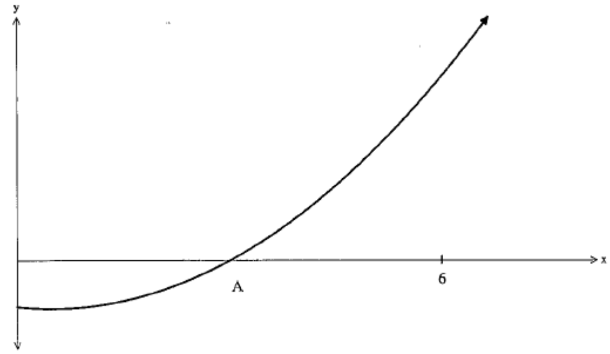


Use the graph to find the value of k which satisfies $\int_{-6}^{k} f(x)\,dx = 0$.

A. 6 B. 10 C. 11 D. 12

Check

6 The diagram below shows the graph of $y = x^2 - x - 6$.



(a) What is the coordinate of A? 1

(b) Find the area bounded by the x-axis and the curve $y = x^2 - x - 6$ for the interval $0 \le x \le 6$. 3

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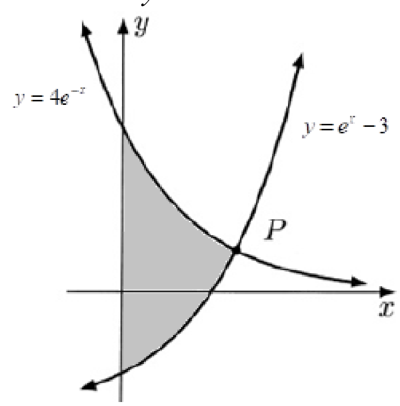
7 The diagram shows the graphs of $y = 4e^{-x}$ and $y = e^x - 3$.



(a) Show that the curves intersect when $e^{2x} - 3e^x - 4 = 0$. 1

(b) Hence, show the x coordinate of the point P is $x = \ln 4$. 2

(c) Find the exact shaded area between the two curves. 2

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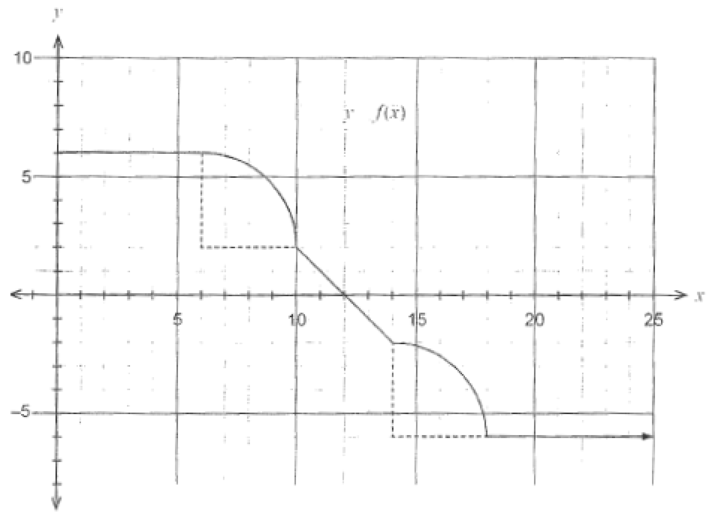
8 Consider the graph $y = f(x)$ given in the diagram. Both arcs have a radius of 4 units.

Using the graph of $y = f(x),\ x \geq 0,$ evalauate exactly the following integrals:

(a) $\displaystyle\int_0^{12} f(x)dx;$ **3**

(b) $\displaystyle\int_0^{18} f(x)dx.$ **2**



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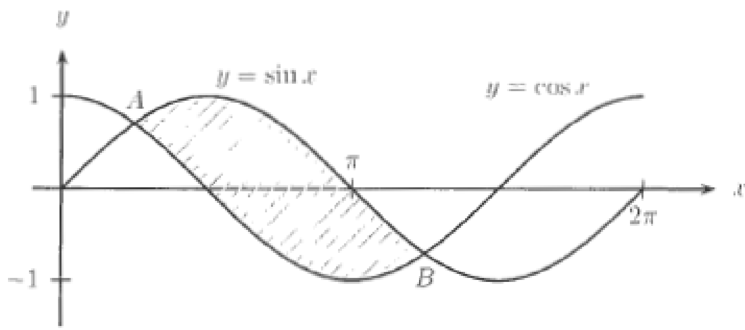
9 The diagram shows the graphs $y = \sin x$ and $y = \cos x,\ 0 \leq x \leq 2\pi$. The graphs intersect at A and B.

(a) Show that A has coordinates $\left(\dfrac{\pi}{4}, \dfrac{1}{\sqrt{2}}\right)$. **2**

(b) Find the area enclosed by the two graphs. **3**



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10 The functions $y=-x^3+4x$ and $y=x$ are sketched below.

(a) Show that the functions intersect when $x=0$ and $x=\pm\sqrt{3}$. **2**

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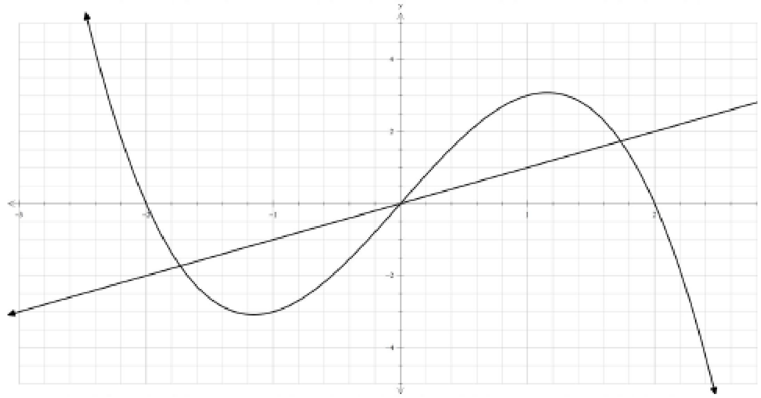
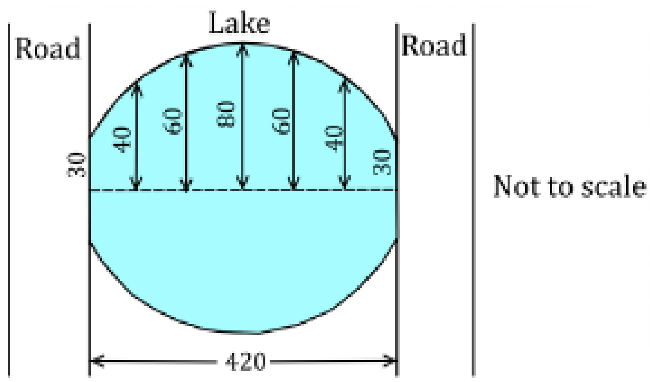


Check

11 A symmetrical lake has two roads, 420 metres apart, forming its sides **3**

Equally spaced measurements of the lake, in metres, are shown on the diagram.

Use the trapezoidal rule to estimate the area of the lake.



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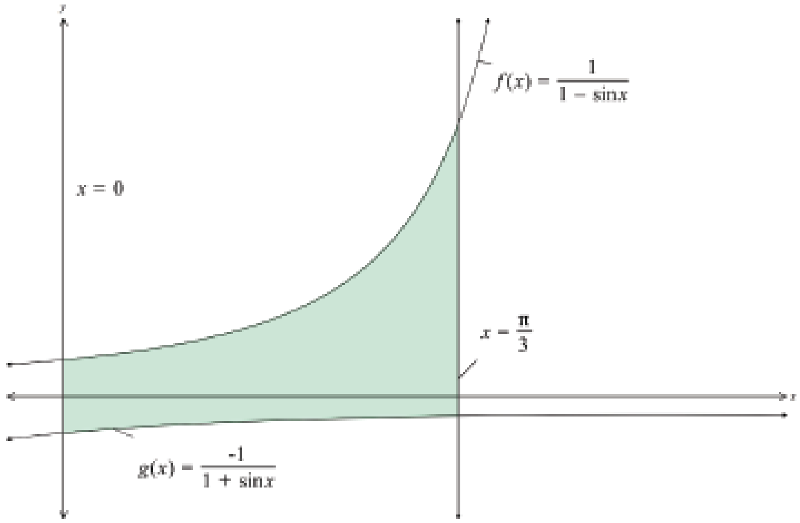
12

(a) Prove that $\dfrac{1}{1-\sin x}+\dfrac{1}{1+\sin x}\equiv 2\sec^2 x$ where $\sin x \neq \pm 1$. **1**

(b) The region shown below represents the area between the curves $f(x)=\dfrac{1}{1-\sin x}$ and $g(x)=\dfrac{-1}{1+\sin x}$, for the domain $\left[0,\dfrac{\pi}{3}\right]$. **3**

Calculate the area between the two curves for the given domain.



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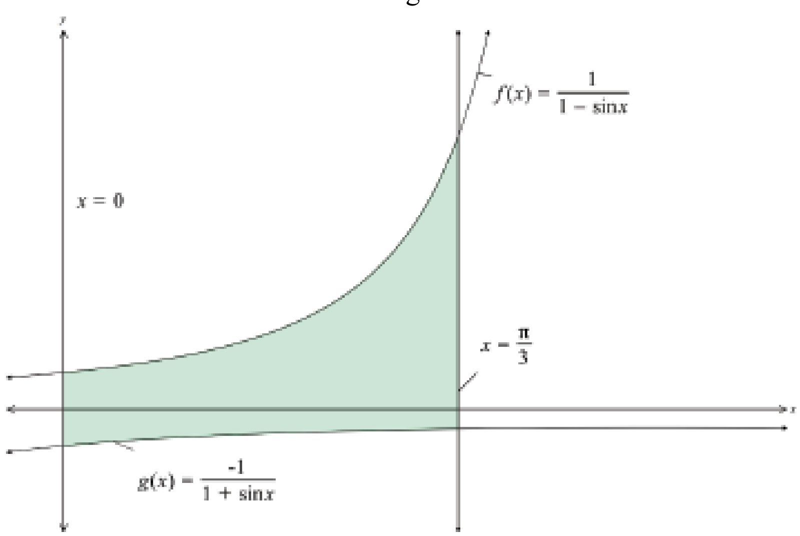

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Check

Solutions 1 2 3 4 5 6 7 8 9 10 11 12 13

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B

$$\int_{1.2}^{1.6} f(x)dx \approx \frac{0.2}{2}(3+7.6+4.8) = \frac{1}{10}\times 15.4 = 1.54$$

Questions 1 2 3 4 5 6 7 8 9 10 11 12 13

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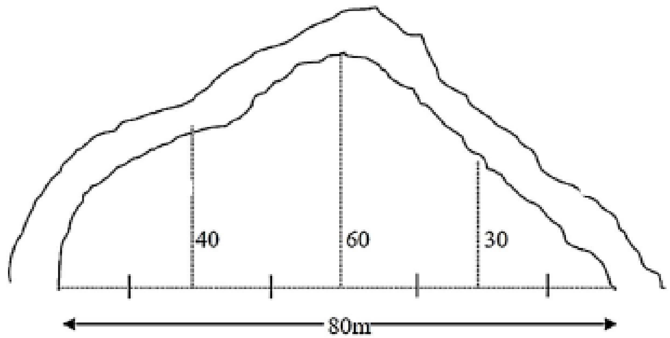

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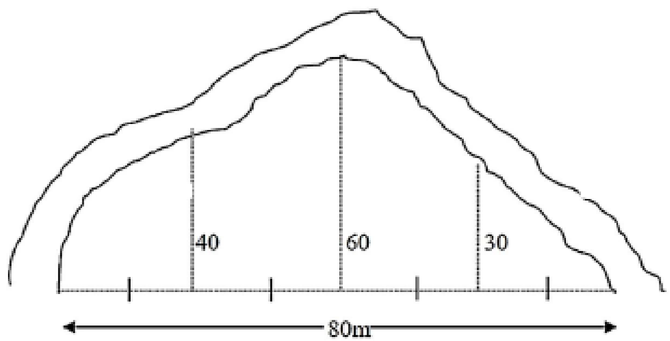

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C

$$A=\frac{20}{2}(0+2(40+60+30)+0)=10\times 260=2600$$

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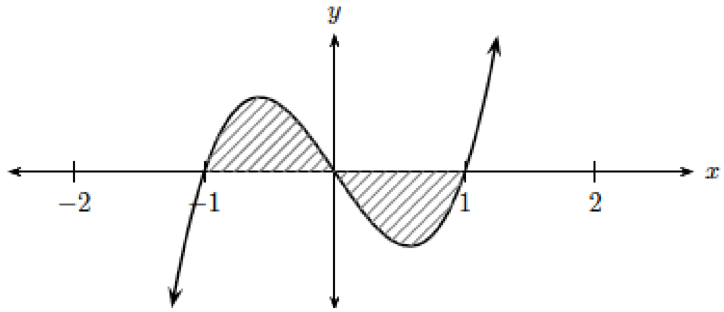

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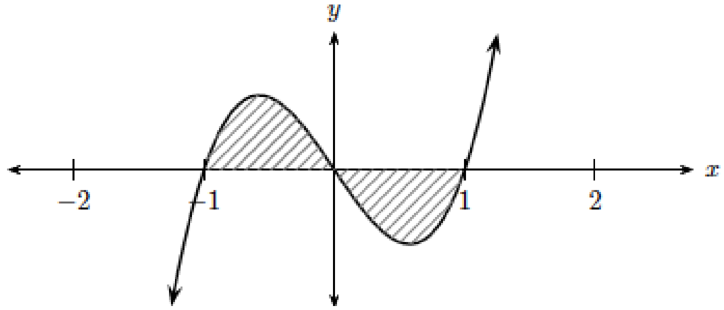

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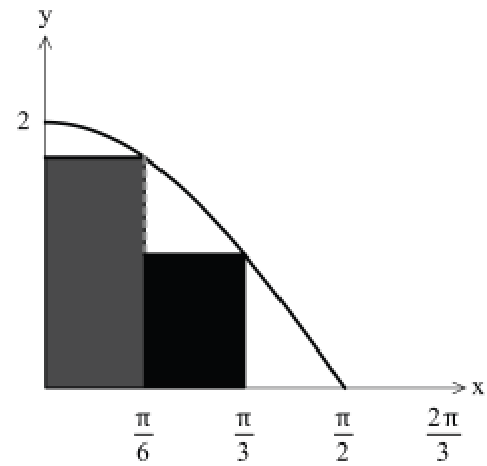
A

The area under the curve $y = 2\cos x$, as shown below, is approximated by two rectangles. **1**



The value of the approximation is:

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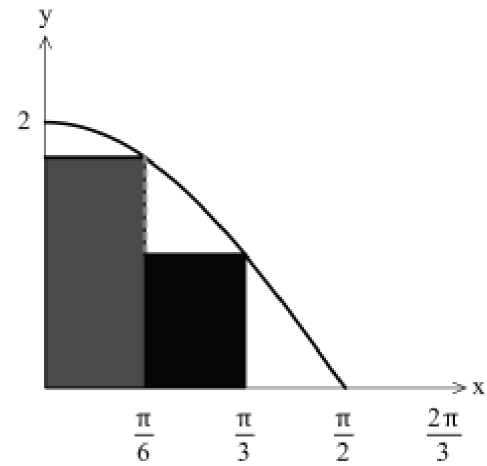

The value of the approximation is:

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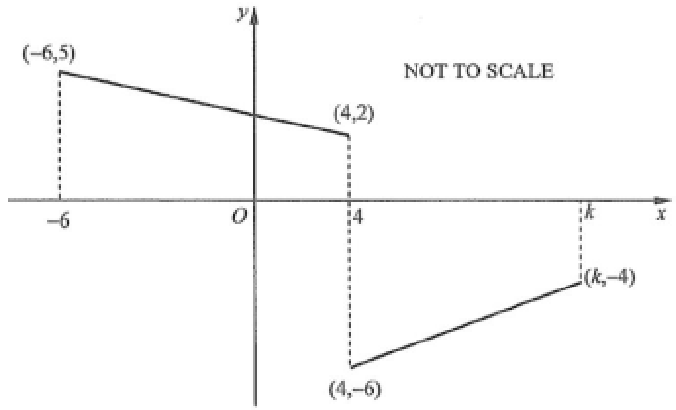
B

$$\text{Area} = \frac{\pi}{6} \times 2\cos\frac{\pi}{6} + \frac{\pi}{6} \times 2\cos\frac{\pi}{3} = \frac{\pi}{6}\left(\sqrt{3}+1\right)$$



1

Use the graph to find the value of k which satisfies $\int_{-6}^{k} f(x)dx = 0$.

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Check

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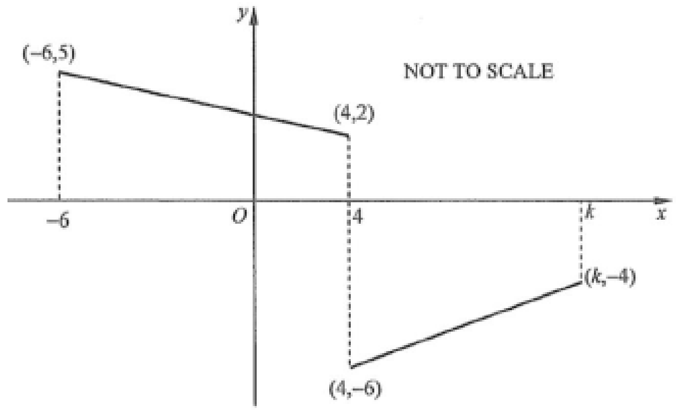

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C

$$\int_{-6}^{4} f(x)dx = \frac{1}{2}(5+2)\times 10 = 35$$

$$\int_{4}^{k} f(x)dx = \frac{1}{2}(-6-4)\times(k-4) = -5(k-4)$$

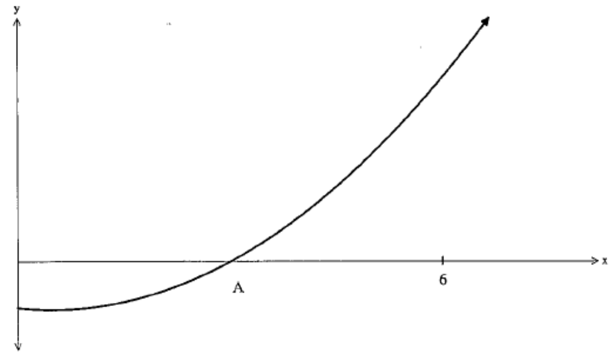
$$35 - 5(k-4) = 0$$

$$5(k-4) = 35$$

$$k-4 = 7$$

$$k = 11$$

The diagram below shows the graph of $y = x^2 - x - 6$.



(a) What is the coordinate of A? **1**

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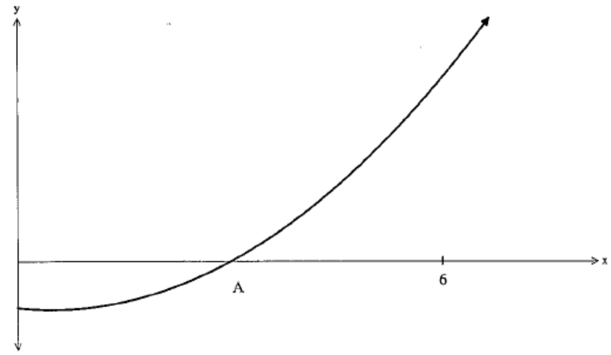

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$y = x^2 - x - 6.$

(a) $A, y = 0: \quad 0 = x^2 - x - 6$

$(x-3)(x+2) = 0$

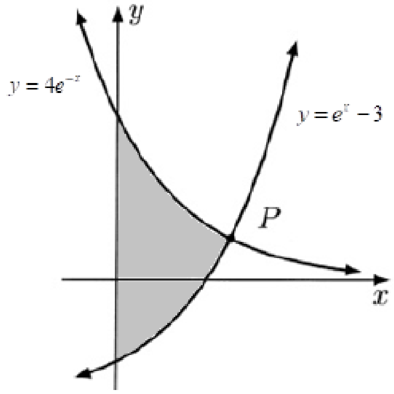
$x = 3, -2$

$x = 3 : A(3,0)$

(b)
$$\text{Area} = \left|\int_0^3 \left(x^2 - x - 6\right) dx\right| + \int_3^6 \left(x^2 - x - 6\right) dx$$
$$= \left|\left[\frac{x^3}{3} - \frac{x^2}{2} - 6x\right]_0^3\right| + \left[\frac{x^3}{3} - \frac{x^2}{2} - 6x\right]_3^6$$
$$= \left|9 - \frac{9}{2} - 18 - 0\right| + 72 - 18 - 36 - \left(9 - \frac{9}{2} - 18\right)$$
$$= \left|-\frac{27}{2}\right| + 18 + \frac{27}{2}$$
$$= 45 \text{ u}^2$$

Questions 1 2 3 4 5 6 7 8 9 10 11 12 13

The diagram shows the graphs of $y = 4e^{-x}$ and $y = e^{x} - 3$.



(a) Show that the curves intersect when $e^{2x} - 3e^{x} - 4 = 0$. **1**

(b) Hence, show the x coordinate of the point P is $x = \ln 4$. **2**

(c) Find the exact shaded area between the two curves. **2**

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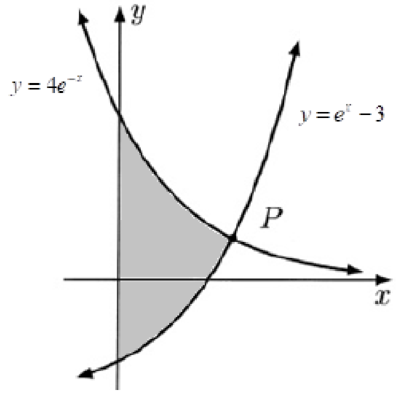

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$y = 4e^{-x}$, $y = e^x - 3$.

(a) Intersect when: $4e^{-x} = e^x - 3$

$4 = e^{2x} - 3e^x$

$e^{2x} - 3e^x - 4 = 0.$

(b) $\left(e^x - 4\right)\left(e^x + 1\right) = 0$

$e^x = 4, -1$. But $e^x > 0$

$x = \ln 4$ IS the x coordinate of the point P.

(c) Area $= \displaystyle\int_0^{\ln 4} \left(4e^{-x} - \left(e^x - 3\right)\right) dx$

$= \displaystyle\int_0^{\ln 4} \left(4e^{-x} - e^x + 3\right) dx$

$= \left[-4e^{-x} - e^x + 3x\right]_0^{\ln 4}$

$= -4e^{-\ln 4} - e^{\ln 4} + 3\ln 4 - \left(-4 - 1 + 0\right)$ $\qquad e^{-\ln 4} = e^{\ln\frac{1}{4}} = \frac{1}{4}$

$= -4 \times \frac{1}{4} - 4 + 3\ln 4 + 5$

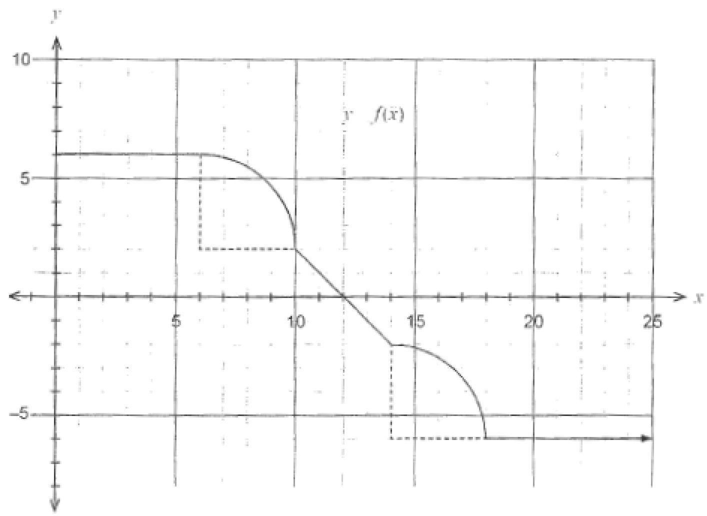
$= 3\ln 4 = 6\ln 2 \text{ u}^2$

Consider the graph $y = f(x)$ given in the diagram. Both arcs have a radius of 4 units.

Using the graph of $y = f(x)$, $x \geq 0$, evalauate exactly the following integrals:

(a) $\int_0^{12} f(x)dx$; **3**

(b) $\int_0^{18} f(x)dx$. **2**



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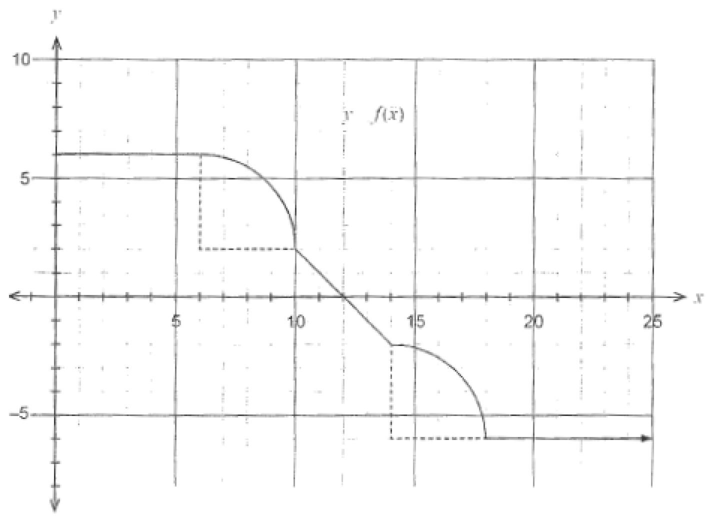


(a) $\displaystyle\int_0^{12} f(x)dx = 6\times 6 + 4\times 2 + \frac{1}{4}\pi\times 4^2 + \frac{1}{2}\times 2\times 2$

$= 36 + 8 + 4\pi + 2$

$= 46 + 4\pi$

(b) $\displaystyle\int_0^{18} f(x)dx = 46 + 4\pi - \left(\frac{1}{2}\times 2\times 2 + (24 - 4\pi)\right)$

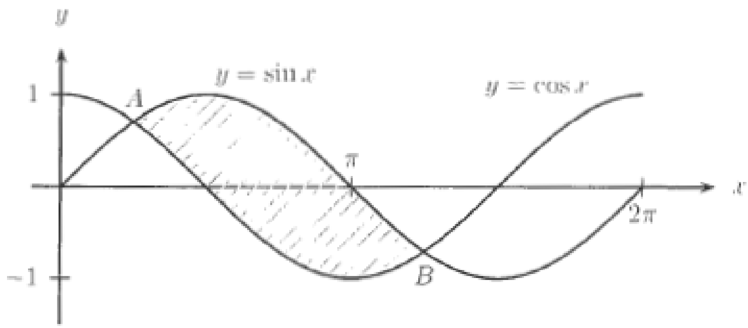
$= 46 + 4\pi - (2 + 24 - 4\pi)$

$= 20 + 8\pi$

The diagram shows the graphs $y = \sin x$ and $y = \cos x$, $0 \le x \le 2\pi$. The graphs intersect at A and B.

(a) Show that A has coordinates $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$. **2**

(b) Find the area enclosed by the two graphs. **3**



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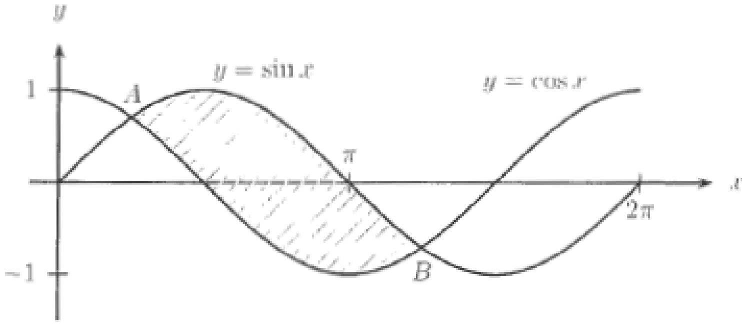

$y = \sin x$, $y = \cos x$, $0 \le x \le 2\pi$.

Intersect at A and B.

(a) $\sin x = \cos x$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$y = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$A\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$

$B\left(\frac{5\pi}{4}, -\frac{1}{\sqrt{2}}\right)$

(b) Area $= \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)dx$

$= \left[-\cos x - \sin x\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$

$= -\left(\cos\frac{5\pi}{4} + \sin\frac{5\pi}{4} - \cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right)$

$= -\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$

$= 2\sqrt{2}\ \text{u}^2$

The functions $y = -x^3 + 4x$ and $y = x$ are sketched below.

(a) Show that the functions intersect when $x = 0$ and $x = \pm\sqrt{3}$. **2**

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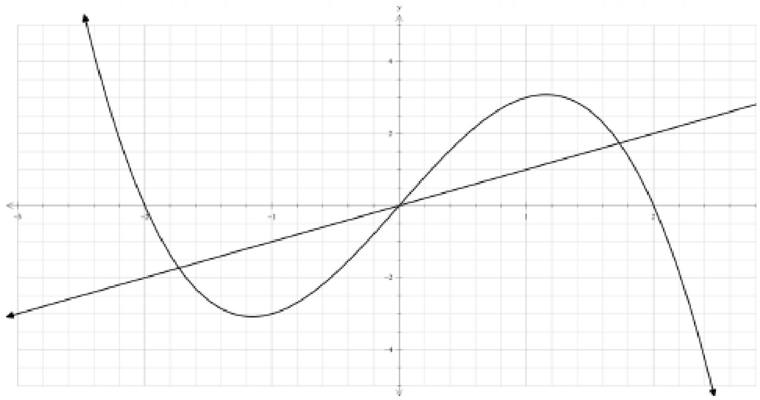

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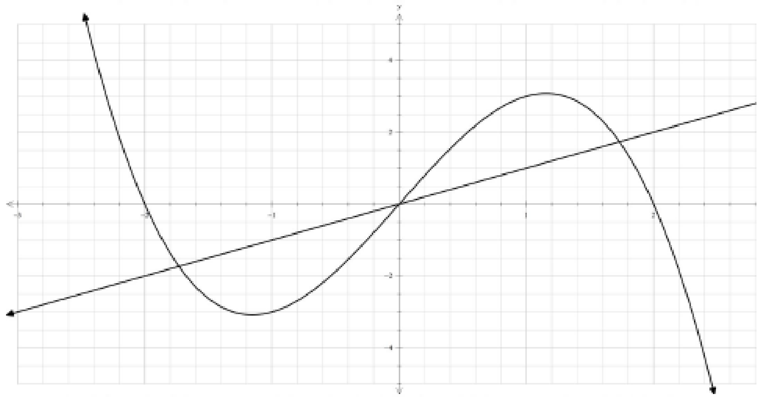


$y = -x^3 + 4x, \;\; y = x$

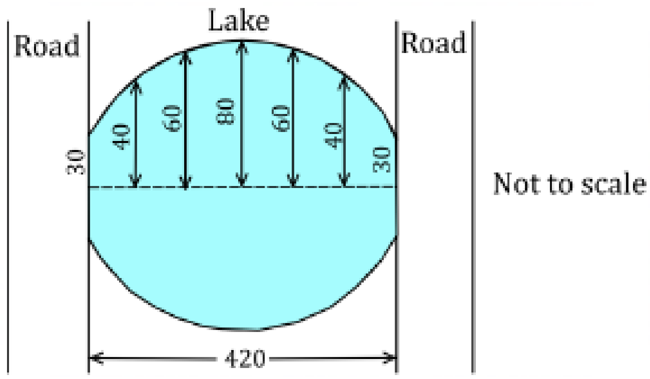
(a)
$$-x^3 + 4x = x$$
$$x^3 - 3x = 0$$
$$x\left(x^2 - 3\right) = 0$$
$$x = 0, \pm\sqrt{3}$$

(b)
$$\text{Area} = \int_0^{\sqrt{3}} \left(-x^3 + 4x - x\right) dx$$
$$= \int_0^{\sqrt{3}} \left(-x^3 + 3x\right) dx$$
$$= \left[-\frac{x^4}{4} + \frac{3x^2}{2}\right]_0^{\sqrt{3}}$$
$$= -\frac{9}{4} + \frac{9}{2} - 0$$
$$= \frac{9}{4} \text{ u}^2$$

A symmetrical lake has two roads, 420 metres apart, forming its sides **3**

Equally spaced measurements of the lake, in metres, are shown on the diagram.

Use the trapezoidal rule to estimate the area of the lake.



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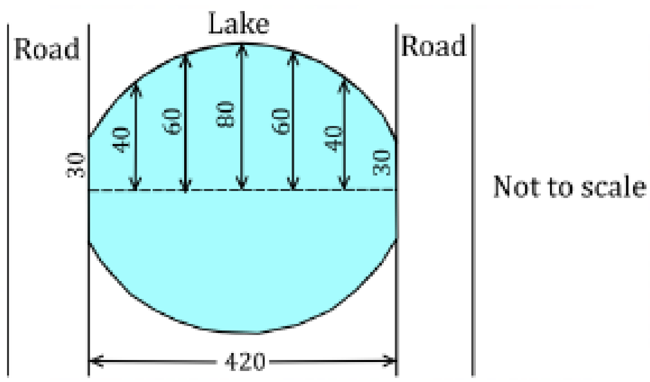


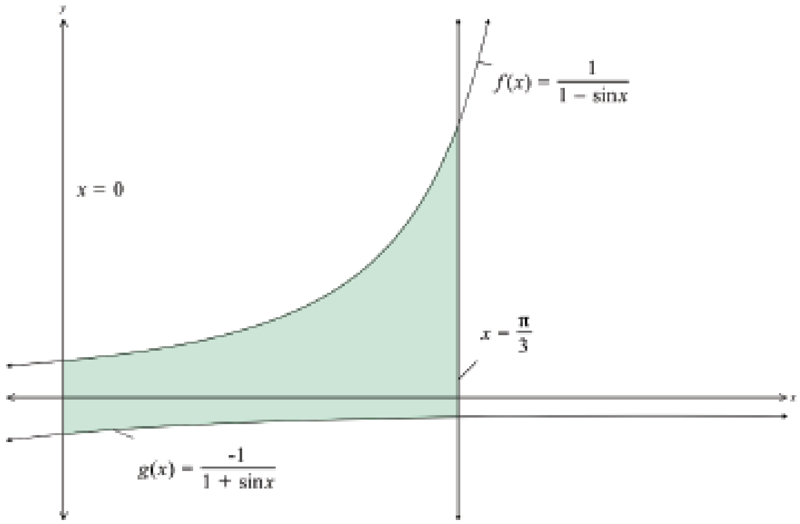
Width of each division $= \dfrac{420}{6} = 70$ m

$$\text{Area of the lake} = 2\times\frac{70}{2}\left(30+2(40+60+80+60+40)+30\right)$$
$$= 70\left(60+2\times 280\right)$$
$$= 43\,400 \text{ m}^2$$

(a) Prove that $\dfrac{1}{1-\sin x}+\dfrac{1}{1+\sin x}\equiv 2\sec^2 x$ where $\sin x \neq \pm 1$. **1**

(b) The region shown below represents the area between the curves $f(x)=\dfrac{1}{1-\sin x}$ and $g(x)=\dfrac{-1}{1+\sin x}$, for the domain $\left[0,\dfrac{\pi}{3}\right]$. **3**

Calculate the area between the two curves for the given domain.



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Solutions 1 2 3 4 5 6 7 8 9 10 11 12 13

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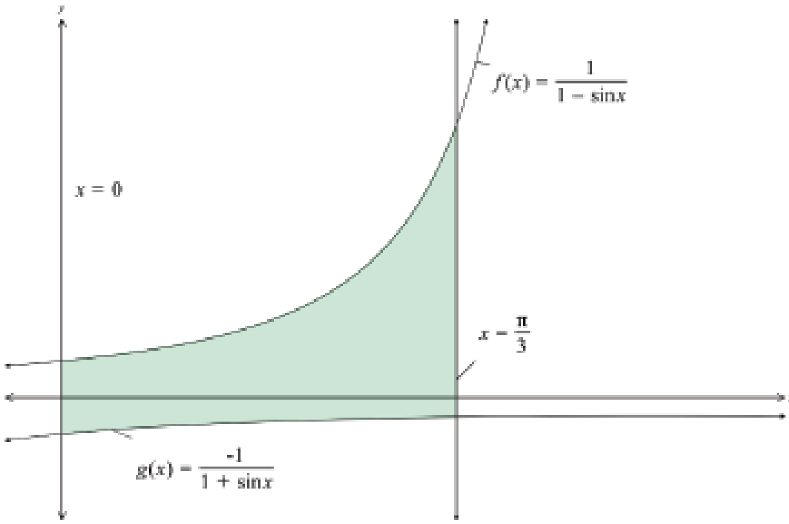
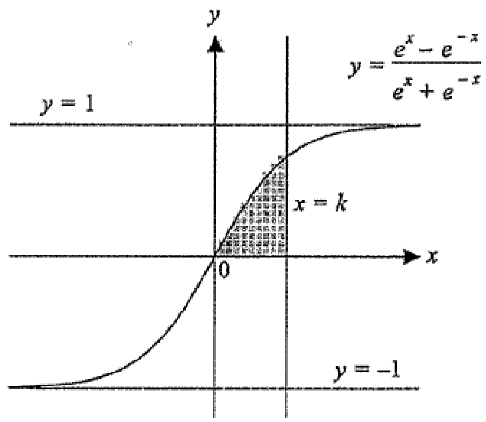


(a)
$$\begin{aligned}\text{Exp} &= \frac{1}{1-\sin x}+\frac{1}{1+\sin x}\\ &= \frac{1+\sin x+1-\sin x}{1-\sin^2 x}\\ &= \frac{2}{\cos^2 x}\\ &= 2\sec^2 x\end{aligned}$$

(b) $f(x)=\dfrac{1}{1-\sin x},\ g(x)=\dfrac{-1}{1+\sin x},\ \left[0,\dfrac{\pi}{3}\right]$

$$\begin{aligned}\text{Area} &= \int_0^{\frac{\pi}{3}}\left(f(x)-g(x)\right)dx\\ &= \int_0^{\frac{\pi}{3}}\left(\frac{1}{1-\sin x}-\frac{-1}{1+\sin x}\right)dx\\ &= \int_0^{\frac{\pi}{3}}\left(\frac{1}{1-\sin x}+\frac{1}{1+\sin x}\right)dx\\ &= \int_0^{\frac{\pi}{3}}\sec^2 x\,dx\\ &= \left[\tan x\right]_0^{\frac{\pi}{3}}\\ &= \tan\frac{\pi}{3}-\tan 0\\ &= \sqrt{3}\ \text{u}^2\end{aligned}$$

Questions 1 2 3 4 5 6 7 8 9 10 11 12 13



The diagram shows the graph of the curve $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

(a) Show that the shaded region bounded by the curve, the x-axis and the line $x = k$, where $k > 0$, has area $\ln\left(\dfrac{e^k + e^{-k}}{2}\right)$. **2**

(b) Find, in simplest exact form, the value of k such that the shaded region has area of 1 square unit. **3**

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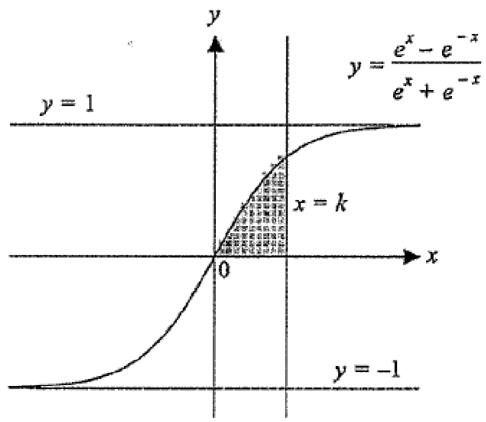

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$y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

(a)
$$\begin{aligned}\text{Area} &= \int_0^k \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx \\ &= \left[\ln\left(e^x + e^{-x}\right)\right]_0^k \\ &= \ln\left(e^k + e^{-k}\right) - \ln(1+1) \\ &= \ln\left(\frac{e^k + e^{-k}}{2}\right)\end{aligned}$$

(b)
$$\begin{aligned}\ln\left(\frac{e^k + e^{-k}}{2}\right) &= 1 \\ \frac{e^k + e^{-k}}{2} &= e \\ e^k + e^{-k} &= 2e \\ e^{2k} + 1 &= 2e^{k+1} \\ e^{2k} - 2e.e^k + 1 &= 0 \\ e^k &= \frac{2e \pm \sqrt{4e^2 - 4}}{2} \\ e^k &= e \pm \sqrt{e^2 - 1} \\ k &= \ln\left(e \pm \sqrt{e^2 - 1}\right)\end{aligned}$$