MX SQ **15** Points P and A are on the number plane. **4**

The vector is $\overrightarrow{PA}$ is $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

Point B is chosen so that the area of $\triangle PAB$ is 10 square units and $|\overrightarrow{PB}| = 4\sqrt{5}$.

Find all possible vectors $\overrightarrow{PB}$.

$|\overrightarrow{PA}| = \sqrt{3^2 + 1^2} = \sqrt{10}$

Let $\angle PAB = \theta$.

$\therefore$ Area $\triangle PAB = \frac{1}{2} \times \sqrt{10} \times 4\sqrt{5} \times \sin\theta = 10$

$\therefore 10\sqrt{2}\sin\theta = 10$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

Hence, $\cos\theta = \pm\frac{1}{\sqrt{2}}$

Let $\overrightarrow{PB} = \begin{pmatrix} x \\ y \end{pmatrix}$.

Also, as $\overrightarrow{PA} . \overrightarrow{PB} = |\overrightarrow{PA}|.|\overrightarrow{PB}|.\cos\theta$:

$\begin{pmatrix} 3 \\ 1 \end{pmatrix} . \begin{pmatrix} x \\ y \end{pmatrix} = \sqrt{10} \times 4\sqrt{5} \times \pm\frac{1}{\sqrt{2}}$

$3x + y = \pm 20$

Also, $|\overrightarrow{PB}| = \sqrt{x^2 + y^2} = 4\sqrt{5}$

$x^2 + y^2 = 80$

Using $3x + y = 20$

$\therefore y = 20 - 3x$ ....(1)

$x^2 + y^2 = 80$ ....(2)

Subs (1) into (2):

$x^2 + (20 - 3x)^2 = 80$

$x^2 + 400 - 120x + 9x^2 = 80$

$10x^2 - 120x + 320 = 0$

$x^2 - 12x + 32 = 0$

$(x - 8)(x - 4) = 0$

$x = 8$ or 4

Substituting into (1):

$x = 8$ and $y = -4$ or $x = 4$ and $y = -8$.

Also, using $3x + y = -20$

$\therefore y = -20 - 3x$ ....(3)

$x^2 + y^2 = 80$ .... (4)

Subs (3) into (4):

$x^2 + (-20 - 3x)^2 = 80$

$x^2 + 400 + 120x + 9x^2 = 80$

$10x^2 + 120x + 320 = 0$

$x^2 + 12x + 32 = 0$

$(x + 8)(x + 4) = 0$

$x = -8$ or -4

Substituting into (3):

$x = -8$ and $y = 4$ or $x = -4$ and $y = -8$.

Possible vectors $\overrightarrow{PB}$ are $\begin{pmatrix} 8 \\ -4 \end{pmatrix}, \begin{pmatrix} 4 \\ -8 \end{pmatrix}, \begin{pmatrix} -8 \\ 4 \end{pmatrix}, \begin{pmatrix} -4 \\ -8 \end{pmatrix}$.