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MX 15 Points P and A are on the number plane.

4

SQ The vector \vec{PA} is $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

Point B is chosen so that the area of $\triangle PAB$ is 10 square units and $|\vec{PB}| = 4\sqrt{5}$.

Find all possible vectors \vec{PB} .

$$|\vec{PA}| = \sqrt{3^2 + 1^2} = \sqrt{10}$$

Let $\angle PAB = \theta$.

$$\therefore \text{Area } \triangle PAB = \frac{1}{2} \times \sqrt{10} \times 4\sqrt{5} \times \sin \theta = 10$$

$$\therefore 10\sqrt{2} \sin \theta = 10$$

$$\sin \theta = \frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$\text{Hence, } \cos \theta = \pm \frac{1}{\sqrt{2}}$$

$$\text{Let } \vec{PB} = \begin{pmatrix} x \\ y \end{pmatrix}.$$

Also, as $\vec{PA} \cdot \vec{PB} = |\vec{PA}| \cdot |\vec{PB}| \cdot \cos \theta$:

$$\begin{pmatrix} 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \sqrt{10} \times 4\sqrt{5} \times \pm \frac{1}{\sqrt{2}}$$

$$3x + y = \pm 20$$

$$\text{Also, } |\vec{PB}| = \sqrt{x^2 + y^2} = 4\sqrt{5}$$

$$x^2 + y^2 = 80$$

$$\text{Using } 3x + y = 20$$

$$\therefore y = 20 - 3x \dots\dots\dots \textcircled{1}$$

$$x^2 + y^2 = 80 \dots\dots\dots \textcircled{2}$$

Subs $\textcircled{1}$ into $\textcircled{2}$:

$$x^2 + (20 - 3x)^2 = 80$$

$$x^2 + 400 - 120x + 9x^2 = 80$$

$$10x^2 - 120x + 320 = 0$$

$$x^2 - 12x + 32 = 0$$

$$(x - 8)(x - 4) = 0$$

$$x = 8 \text{ or } 4$$

Substituting into $\textcircled{1}$:

$$x = 8 \text{ and } y = -4 \text{ or } x = 4 \text{ and } y = -8.$$

Also, using $3x + y = -20$

$$\therefore y = -20 - 3x \dots\dots\dots \textcircled{3}$$

$$x^2 + y^2 = 80 \dots\dots\dots \textcircled{4}$$

Subs $\textcircled{3}$ into $\textcircled{4}$:

$$x^2 + (-20 - 3x)^2 = 80$$

$$x^2 + 400 + 120x + 9x^2 = 80$$

$$10x^2 + 120x + 320 = 0$$

$$x^2 + 12x + 32 = 0$$

$$(x + 8)(x + 4) = 0$$

$$x = -8 \text{ or } -4$$

Substituting into $\textcircled{3}$:

$$x = -8 \text{ and } y = 4 \text{ or } x = -4 \text{ and } y = -8.$$

Possible vectors \vec{PB} are $\begin{pmatrix} 8 \\ -4 \end{pmatrix}, \begin{pmatrix} 4 \\ -8 \end{pmatrix}, \begin{pmatrix} -8 \\ 4 \end{pmatrix}, \begin{pmatrix} -4 \\ -8 \end{pmatrix}$.

* These solutions have been provided by [projectmaths](#) and are not supplied or endorsed by NESA.

