TG 3 Let A and B be points with position vectors $\underset{\sim}{a} = \begin{pmatrix}3\\1\end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix}3\\2\end{pmatrix}$ respectively.

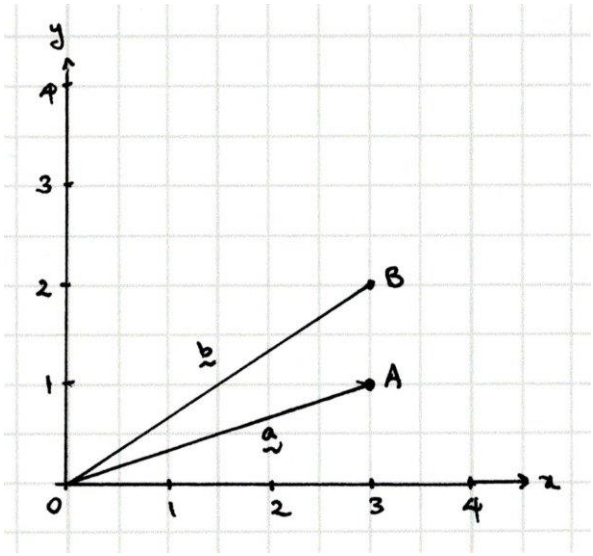
(a) Draw a diagram showing the points O, A and B.

(b) Calculate the angle AOB

(i) by finding the tangents of the angles α and β between $\underset{\sim}{a}$ and the unit vector $\underset{\sim}{i}$, and $\underset{\sim}{b}$ and the unit vector $\underset{\sim}{j}$, and using the formula for $\tan(\alpha - \beta)$.

(ii) by using a method based on scalar products.

(a)



(b)(i) Let $\angle BOx = \alpha$

$\therefore \tan\alpha = \frac{2}{3}$

Let $\angle AOx = \beta$

$\therefore \tan\beta = \frac{1}{3}$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$= \frac{\frac{2}{3} - \frac{1}{3}}{1 + \frac{2}{3}\times\frac{1}{3}}$$

$$= \frac{3}{11}$$

$\therefore \angle AOB = \tan^{-1}\frac{3}{11} = 15^\circ\ 15'$ (nearest min)

(ii) $\left|\underset{\sim}{a}\right| = \sqrt{3^2 + 1^2}$

$= \sqrt{10}$

$\left|\underset{\sim}{b}\right| = \sqrt{3^2 + 2^2}$

$= \sqrt{13}$

$$\cos\angle AOB = \frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\left|\underset{\sim}{a}\right|\left|\underset{\sim}{b}\right|}$$

$$= \frac{3\times 3 + 1\times 2}{\sqrt{10}\times\sqrt{13}}$$

$$= \frac{11}{\sqrt{130}}$$

$\angle AOB = 15^\circ\ 15'$ (nearest min)