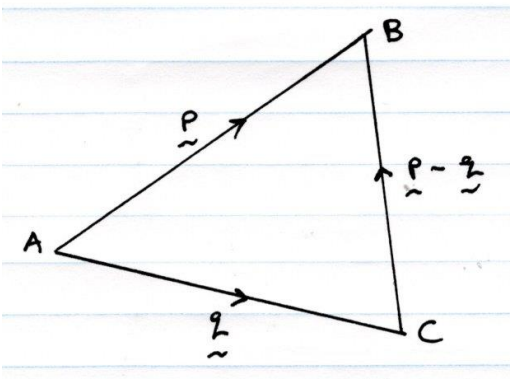
**TG 6** In a triangle $ABC$ denote the displacement vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ by $\underset{\sim}{p}$ and $\underset{\sim}{q}$ respectively.

Use vectors to prove that, in a triangle $ABC$, $a^2 = b^2 + c^2 - 2bc\cos A$.

Need to prove $|\underset{\sim}{p} - \underset{\sim}{q}|^2 = |\underset{\sim}{p}|^2 + |\underset{\sim}{q}|^2 - 2|\underset{\sim}{p}||\underset{\sim}{q}|\cos A$ .




$$\text{RHS} = |\underset{\sim}{p}|^2 + |\underset{\sim}{q}|^2 - 2|\underset{\sim}{p}||\underset{\sim}{q}|\frac{\underset{\sim}{p}\cdot\underset{\sim}{q}}{|\underset{\sim}{p}||\underset{\sim}{q}|}$$

$$= |\underset{\sim}{p}|^2 + |\underset{\sim}{q}|^2 - 2\,\underset{\sim}{p}\cdot\underset{\sim}{q}$$

$$= |\underset{\sim}{p}|^2 - 2\,\underset{\sim}{p}\cdot\underset{\sim}{q} + |\underset{\sim}{q}|^2$$

$$= |\underset{\sim}{p} - \underset{\sim}{q}|^2$$

$$= \text{LHS}$$

$$\therefore\ |\underset{\sim}{p} - \underset{\sim}{q}|^2 = |\underset{\sim}{p}|^2 + |\underset{\sim}{q}|^2 - 2|\underset{\sim}{p}||\underset{\sim}{q}|\cos A$$

$$\therefore\ a^2 = b^2 + c^2 - 2bc\cos A$$