TG 6
In a triangle $A B C$ denote the displacement vectors $\overrightarrow{A B}$ and $\overrightarrow{A C}$ by $\underset{\sim}{p}$ and $\underset{\sim}{q}$ respectively. Use vectors to prove that, in a triangle $A B C, a^{2}=b^{2}+c^{2}-2 b c \cos A$.
Need to prove $|\underset{\sim}{p}-\underset{\sim}{q}|^{2}=|\underset{\sim}{p}|^{2}+|\underset{\sim}{q}|^{2}-2|\underset{\sim}{p}||\underset{\sim}{q}| \cos A$.

$$
\begin{aligned}
& \text { RHS }=|\underset{\sim}{p}|^{2}+|\underset{\sim}{q}|^{2}-2|\underset{\sim}{p}||\underset{\sim}{q}| \underset{\sim}{p} \underset{\sim}{p}|\underset{\sim}{p}| \underset{\sim}{q}| |(\underset{\sim}{q} \mid \\
& =|\underset{\sim}{p}|^{2}+|\underset{\sim}{q}|^{2}-2 \underset{\sim}{p} \cdot \underset{\sim}{q} \\
& =|\underset{\sim}{p}|^{2}-2 \underset{\sim}{p \cdot q} \underset{\sim}{q}+\left.\underset{\sim}{\mid q}\right|^{2} \\
& =|p-q|^{2} \\
& \text { = LHS }
\end{aligned}
$$

$\therefore|\underset{\sim}{p}-\underset{\sim}{q}|^{2}=|\underset{\sim}{p}|^{2}+|\underset{\sim}{q}|^{2}-2|\underset{\sim}{p}||\underset{\sim}{q}| \cos A$
$\therefore a^{2}=b^{2}+c^{2}-2 b c \cos A$

* These solutions have been provided by projectmaths and are not supplied or endorsed by NESA.

