TG 7

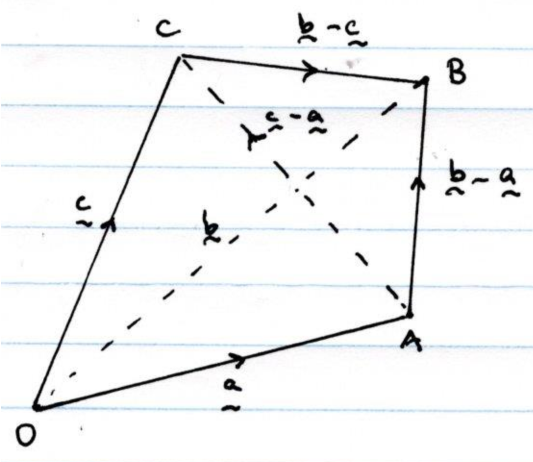
Let $OABC$ be a kite, with OB as its line of symmetry. Let $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$ and $\overrightarrow{OC} = \underset{\sim}{c}$.

(a) Write the vectors $\overrightarrow{AB}$ and $\overrightarrow{CB}$ in terms of $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$.

(b) Using the fact that the lengths of AB and CB are equal, write an equation involving scalar products.

(c) Use this equation to prove that the diagonals of a kite are perpendicular.

(a)



$\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$

$\overrightarrow{CB} = \underset{\sim}{b} - \underset{\sim}{c}$

(b) $|\underset{\sim}{b} - \underset{\sim}{a}| = |\underset{\sim}{b} - \underset{\sim}{c}|$ and $|\underset{\sim}{a}| = |\underset{\sim}{c}|$

Now, $\overrightarrow{AC} = \underset{\sim}{c} - \underset{\sim}{a}$

$\overrightarrow{AC} \bullet \overrightarrow{OB} = (\underset{\sim}{c} - \underset{\sim}{a}) \bullet \underset{\sim}{b}$

(c) $\overrightarrow{AC} \bullet \overrightarrow{OB} = \underset{\sim}{c} \bullet \underset{\sim}{b} - \underset{\sim}{a} \bullet \underset{\sim}{b}$

$= |\underset{\sim}{c}|\,|\underset{\sim}{b}| \cos \angle COB - |\underset{\sim}{a}|\,|\underset{\sim}{b}| \cos \angle AOB$

$= |\underset{\sim}{a}|\,|\underset{\sim}{b}| \cos \angle COB - |\underset{\sim}{a}|\,|\underset{\sim}{b}| \cos \angle AOB$

As OB is axis of symmetry, then $\angle COB = \angle AOB$.

$\therefore \overrightarrow{AC} \bullet \overrightarrow{OB} = 0$

Hence, $\overrightarrow{AC} \perp \overrightarrow{OB}$.

The diagonals are perpendicular.

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by NESA.