TG 2 A manufacturer of jam jars knows that 8% of the jars produced are defective. He supplies jars in cartons containing 12 jars. He supplies cartons of jars in crates of 60 cartons. In each case, making clear the distribution that you are using, calculate the probability that:

(a) a carton contains exactly two defective jars.
(b) a carton contains at least one defective jar.
(c) a crate contains between 39 and 44 (inclusive) cartons with at least one defective jar.

Projectmaths has provided this probability table extract:

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
…	…	…	…	…	…	…	…	…	…	…
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633

(a) Using Binomial prob: $X \sim \text{Bin}(60, 0.6323)$:

$$P(X = 2) = {}^{12}C_2\,(0.08)^2(0.92)^{10}$$
$$= 0.1835 \text{ (4 dec pl)}$$

(b) Using Binomial probability:

$$P(X \geq 1) = 1 - P(X = 0)$$
$$= 1 - {}^{12}C_0\,(0.08)^0(0.92)^{12}$$
$$= 0.6323 \text{ (4 dec pl)}$$

(c) $n = 60$

$P(\text{defective}) = p = 0.6323$

$np = 60(0.6323) = 37.9\ldots > 10$

$nq = 60(0.3677) = 22.0\ldots > 10$

As $np > 10$ and $nq > 10$, then use normal distribution: $Y \sim N(60, 0.6323)$:

$$\mu_{\hat{p}} = p = 0.6323$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$
$$= \sqrt{\frac{0.6323(1-0.6323)}{60}}$$
$$= 0.0622 \text{ (4 dec pl)}$$

Now, 39 out of 60 is 0.65.

$$z = \frac{x-\mu}{\sigma}$$
$$= \frac{0.65-0.6323}{0.0622}$$
$$= 0.28 \text{ (2 dec pl)}$$

For $z = 0.28$, the table provides 0.6103.

Also, 44 out of 60 is 0.7333 (4 dec pl).

$$z = \frac{x-\mu}{\sigma}$$
$$= \frac{0.7333-0.6323}{0.0622}$$
$$= 1.62 \text{ (2 dec pl)}$$

For $z = 1.62$, the table provides 0.9474.

Hence, $P(39 \leq Y \leq 44) = 0.9474 - 0.6103$
$= 0.3307$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by NESA.