TG **3** A fair coin is tossed 18 times. Use the binomial distribution to find the probability of obtaining 14 Heads. Then use the normal distribution to find the probability of obtaining 14 Heads, and to find the probability of obtaining 14 or more Heads.

Show that the approximation is valid. Projectmaths has provided this probability table extract:

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952

Using Binomial distribution: $X \sim \text{Bin}(18, 0.5)$:

$$P(X = 14) = {}^{18}C_{14}(0.5)^{14}(0.5)^4$$
$$= 0.011672973\ldots$$
$$= 0.0117 \text{ (4 dec pl)}$$

Using normal distribution: $Y \sim N(18, 0.5)$:

$n = 18$ and $p = 0.5$

$$np = 18 \times 0.5$$
$$= 9$$

Also, $nq = 9$.

So, $np < 10$ and $nq < 10$.

$$\sigma = \sqrt{np(1-p)}$$
$$= \sqrt{9(1-0.5)}$$
$$= 2.1213 \text{ (4 dec pl)}$$

For $P(Y = 14)$, consider $P(13.5 \le Z \le 14.5)$:

For 13.5: $$z = \frac{X-\mu}{\sigma}$$
$$= \frac{13.5-9}{2.1213}$$
$$= 2.12 \text{ (2 dec pl)}$$

From the table, $z = 2.12$ gives 0.9830.

For 14.5: $$z = \frac{14.5-9}{2.1213}$$
$$= 2.59 \text{ (2 dec pl)}$$

From the table, $z = 2.59$ gives 0.9952.

$$P(13.5 \le Z \le 14.5) = 0.9952 - 0.9830$$
$$= 0.0122$$

For $P(Y \ge 14)$: For 14.5: $$z = \frac{14-9}{2.1213}$$
$$= 2.36 \text{ (2 dec pl)}$$

From the table, $z = 2.36$ gives 0.9909.

For $P(Y \ge 14) = 1 - 0.9909$
$$= 0.0091$$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by NESA.