TG 7 Find the probability of obtaining 4, 5, 6 or 7 Heads when a fair coin is tossed 12 times

(a) using the binomial theorem.

(b) using a normal approximation to the binomial distribution.

Projectmaths has provided this probability table extract:

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
…	…	…	…	…	…	…	…	…	…	…
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319

(a) Using Binomial distribution:

$P(X = 4 \text{ or } X = 5 \text{ or } X = 6 \text{ or } X = 7)$

$= {}^{12}C_4(0.5)^4(0.5)^8 + {}^{12}C_5(0.5)^5(0.5)^7$

$+ {}^{12}C_6(0.5)^6(0.5)^6 + {}^{12}C_7(0.5)^7(0.5)^5$

$= 0.7332$ (4 dec pl)

(b) Using Normal distribution:

$n = 12$

$P(\text{head}) = p = 0.5$

$np = 12 \times 0.5$

$= 6$ (NB: $np < 10$)

$\sigma = \sqrt{np(1-p)}$

$= \sqrt{6(1-0.5)}$

$= 1.7321$ (4 dec pl)

As $np < 10$, use continuity correction:

$P(3.5 \le X \le 7.5)$

Consider $X = 3.5$: $z = \dfrac{3.5-6}{1.7321}$

$= -1.44$ (2 dec pl)

From the table, $z = 1.44$ gives 0.9251,

so -1.44 gives $1 - 0.9251 = 0.0749$.

Consider $X = 7.5$: $z = \dfrac{7.5-6}{1.7321}$

$= 0.87$ (2 dec pl)

From the table, $z = 0.87$ gives 0.8078.

Hence, P(4, 5, 6 or 7) $= 0.8078 - 0.0749$

$= 0.7329$

* These solutions have been provided by *projectmaths* and are not supplied or endorsed by NESA.