TG PC	**9**	Two players *A* and *B* play a series of games against each other to get a prize. In any game, either of the players is equally likely to win. To begin with, the first player who wins a total of 5 games gets the prize.
15 X	**14 c**	(i) Explain why the probability of player *A* getting the prize in exactly 7 games is $\binom{6}{4}\left(\frac{1}{2}\right)^7$. **1**
		(ii) Write an expression for the probability of player *A* getting the prize in at most 7 games. **1**
		(iii) Suppose now that the prize is given to the first player to win a total of $(n + 1)$ games, where n is a positive integer. By considering the probability that *A* gets the prize, prove that $\binom{n}{n}2^n + \binom{n+1}{n}2^{n-1} + \binom{n+2}{n}2^{n-2} + \ldots + \binom{2n}{n}2^n = 2^{2n}$. **2**

(i) The seventh game is won by player *A*. This means that *A* wins 4 of the first 6 games.

Let p = P(*A* not wins) = $\frac{1}{2}$ Let q = P(*A* wins) = $\frac{1}{2}$

$$\text{P}(A \text{ wins 4 of first 6}) = \binom{6}{4}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^4$$

$$\therefore \text{P}(A \text{ wins 4 of first 6 games and the 7}^{\text{th}} \text{ game}) = \binom{6}{4}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^4 \times \frac{1}{2}$$

$$= \binom{6}{4}\left(\frac{1}{2}\right)^7$$

State Mean: **0.19**

(ii) $$\text{P(prize after 5, 6 or 7 games)} = \binom{4}{4}\left(\frac{1}{2}\right)^5 + \binom{5}{4}\left(\frac{1}{2}\right)^6 + \binom{6}{4}\left(\frac{1}{2}\right)^7$$

State Mean: **0.23**

(iii) If a prize after $(n + 1)$ games, then there is a possible $2n$ number of games before a player wins the prize. Also, [as P(A wins) = P(*B* wins) = $\frac{1}{2}$]

$$\text{P}(A \text{ eventually wins the prize}) = \binom{n}{n}\left(\frac{1}{2}\right)^{n+1} + \binom{n+1}{n}\left(\frac{1}{2}\right)^{n+2} + \ldots + \binom{2n}{n}\left(\frac{1}{2}\right)^{2n+1} = \frac{1}{2}$$

Now as $2^{-1} = \frac{1}{2}$ and $2^{-1} \times 2^{2n+1} = 2^{2n}$, then multiply both sides by 2^{2n+1}:

$$\therefore \binom{n}{n}2^n + \binom{n+1}{n}2^{n-1} + \binom{n+2}{n}2^{n-2} + \ldots + \binom{2n}{n}2^n = 2^{2n}$$

State Mean: **0.17**

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