![projectmaths logo]

SAMPLE ONLY
MATHEMATICS ADVANCED
HSC Exam* Questions by Topic
2023 - 2019

v2024

Licensed from NESA

Year 11 Course

Functions

F1.1 Algebraic techniques

F1.2 Introduction to functions

F1.3 Linear, quadratic & cubic functions

F1.4 Further functions & relations

Trigonometric Functions

T1.1 Trigonometry

T1.2 Radians

T2 Trigonometric functions & identities

Calculus

C1.1 Gradients of tangents

C1.2 Difference quotients

C1.3 The derivative function & its graph

C1.4 Calculating with derivatives

Exponential & Logarithmic Functions

E1.1 Introducing logarithms

E1.2 Logarithmic laws & applications

E1.3 Exponential function & natural logs

E1.4 Graphs & apps of exp & log functions

Statistical Analysis

S1.1 Probability & Venn diagrams

S1.2 Discrete probability distributions

Year 12 Course

Functions

F2 Graphing techniques

Trigonometric Functions

T3 Trig functions and graphs

Calculus

C2.1 Diff of trig, exp & log fns

C2.2 Rules of differentiation

C3.1 The first & second derivs

C3.2 Applications of the deriv

C4.1 The anti-derivative

C4.2 Areas & the definite integral

Financial Mathematics

M1.1 Modelling investments & loans

M1.2 Arithmetic sequences & series

M1.3 Geometric sequences & series

M1.4 Financial apps of sequences & series

Statistical Analysis

S2.1 Data and summary statistics

S2.2 Bivariate data analysis

S3.1 Continuous random variables

S3.2 The normal distribution

Complete Papers

2023 HSC

2022 HSC

2021 HSC

2020 HSC

2020 NESA Sample

Question Difficulty

Easy

Mid-range

Difficult

Mathematics Advanced, Ext 1, Ext 2 Reference Sheet (2023 HSC)

projectmaths

Year 11: Trigonometric Functions
T1.1 Trigonometry

Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017

Students:

- use the sine, cosine and tangent ratios to solve problems involving right-angled triangles where angles are measured in degrees, or degrees and minutes 🔗
- establish and use the sine rule, cosine rule and the area of a triangle formula for solving problems where angles are measured in degrees, or degrees and minutes **AAM** 🔗🖥
- find angles and sides involving the ambiguous case of the sine rule
  - use technology and/or geometric construction to investigate the ambiguous case of the sine rule when finding an angle, and the condition for it to arise ✷ 🖥
- solve problems involving the use of trigonometry in two and three dimensions **AAM** 🔗
  - interpret information about a two or three-dimensional context given in diagrammatic or written form and construct diagrams where required
- solve practical problems involving Pythagoras' theorem and the trigonometry of triangles, which may involve the ambiguous case, including finding and using angles of elevation and depression and the use of true bearings and compass bearings in navigation **AAM** 🔗 ✷

S T A N D A

S T A N D A R D

Reference Sheet

| 23 MA | 16 | The diagram shows a shape *APQBCD*. The shape consists of a rectangle *ABCD* with an arc *PQ* on side *AB* and with side lengths *BC* = 3.6 m and *CD* = 8.0 m. The arc PQ is an arc of a circle with centre O and radius 2.1 m and ∠POQ = 110°. What is the perimeter of the shape APQBCD? Give your answer correct to one decimal place. |

Solution

4
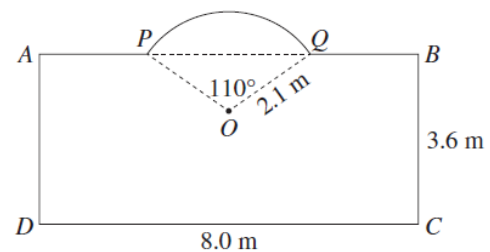
NOT TO SCALE

COMMON QUESTION: NESA 2023 Mathematics Standard 2 and Advanced HSC Examinations

| 23 MA | 22 | In the rectangular prism shown, *AD* = 7 cm, *AE* = 8 cm, *EF* = 6 cm. Point *M* is the midpoint of *CD*. Find ∠*AEM*, to the nearest degree. |

Solution

3
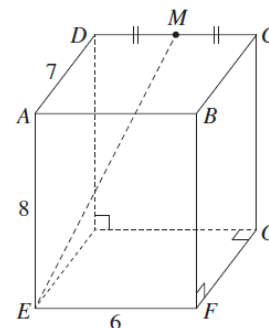
NOT TO SCALE

NESA 2023 Mathematics Advanced HSC Examination

| 23 MS 2 | 27 | The diagram shows the location of three places X, Y and C.
Y is on a bearing of 120° and 15 km from X.
C is 40 km from X and lies due west of Y.
P lies on the line joining C and Y and is due south of X.
(a) Find the distance from X to P.
(b) What is the bearing of C from X, to the nearest degree? | | 2
2 | Solution |

NOT TO SCALE

120°

40 km

15 km
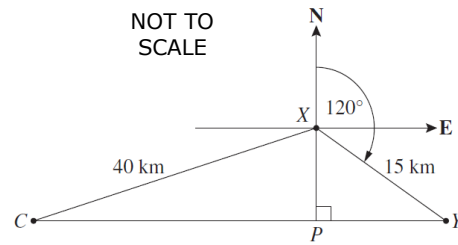
NESA 2023 Mathematics Standard 2 HSC Examination

| 22 MA | 3 | A tower BT has height h metres.
From point A, the angle of elevation to the top of the tower is 26° as shown.
Which of the following is the correct expression for the length of AB?
A. $h \tan 26°$
B. $h \cot 26°$
C. $h \sin 26°$
D. $h \operatorname{cosec} 26°$ | 1 | Solution |

NOT TO SCALE

26°
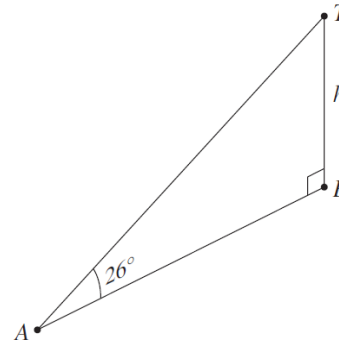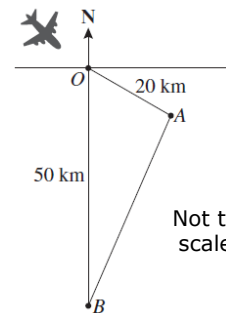
NESA 2022 Mathematics Advanced HSC Examination

| 22 MS 2 | 33 | The diagram shows an aeroplane that was flying towards an airport at A on a bearing of 135°T. When it was at point O, 20 km away from the airport at A, the flight course was changed. The aeroplane landed at an airport at B directly south of O. The distance from O to B is 50 km.
(a) Show that the distance between the airport at A and the airport at B is 38.5 km, correct to 1 decimal place.
(b) Use the sine rule to find the angle OBA to the nearest degree.
(c) What is the bearing of the airport at B from the airport at A? | 2

2

1 | Solution |

20 km

50 km

Not to scale

NESA 2022 Mathematics Standard 2 HSC Examination

| 21 MA | 12 | A right-angled triangle XYZ is cut out from a semicircle with centre O. The length of the diameter XZ is 16 cm and $YXZ = 30°$, as shown on the diagram.
(a) Find the length of XY in cm, correct to two decimal places.

(b) Hence, find the area of the shaded region in square centimetres, correct to one decimal place. | 2
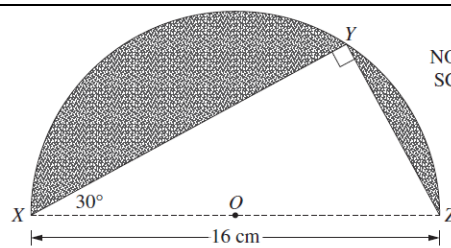
3 | Solution |

NOT TO SCALE

30°

16 cm

| 21 MA | 18 | The diagram shows a triangle ABC where $AC = 25$ cm, $BC = 16$ cm, $BAC = 28°$ and angle ABC is obtuse.

Find the size of the obtuse angle ABC correct to the nearest degree. | 3 | Solution |
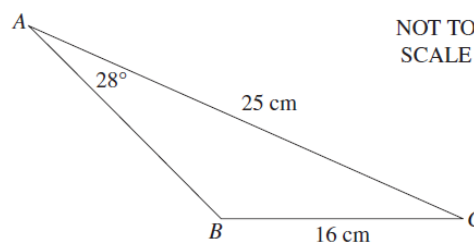
NOT TO SCALE

28°

25 cm

16 cm

21 MS 2	39	The diagram shows a compass radial survey of the field *ABCD*. (a) Triangle *COB* has an area of 466 m². Find the size of acute angle *COB*, correct to the nearest degree. (b) A farmer wants to put a fence around the triangle *DOC*. Find the length of fencing required. Give your answer in metres correct to one decimal place.		2 3	Solution

NESA 2021 Mathematics Standard 2 HSC Examination

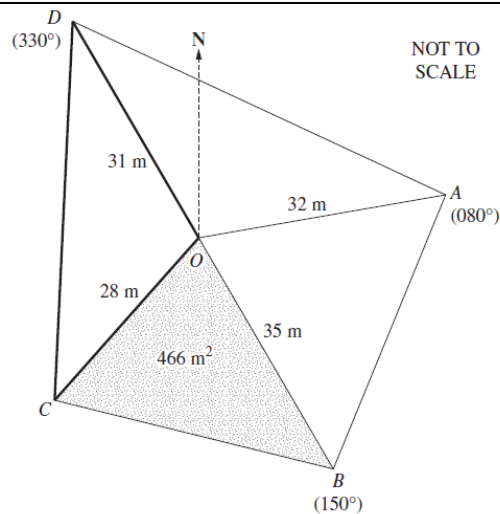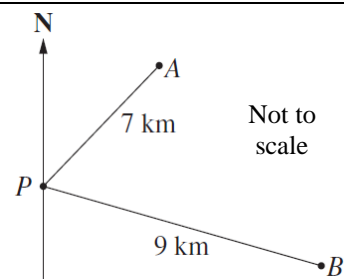| 20 MA	15	Mr Ali, Ms Brown and a group of students were camping at the site located at *P*. Mr Ali walked with some of the students on a bearing of 035° for 7 km to location *A*. Ms Brown, with the rest of the students, walked on a bearing of 100° for 9 km to location *B*. (a) Show that the angle *APB* is 65°. (b) Find the distance *AB*. (c) Find the bearing of Ms Brown's group from Mr Ali's group. Give your answer correct to the nearest degree.		 1 2 2	Solution

COMMON QUESTION: NESA 2020 Mathematics Standard 2 and Advanced HSC Examinations

20 MA	22	The diagram shows a regular decagon (ten-sided shape with all sides equal and all interior angles equal). The decagon has centre *O*. The perimeter of the shape is 80 cm. By considering triangle *OAB*, calculate the area of the ten-sided shape. Give your answer in square centimetres, correct to one decimal place.		4	Solution
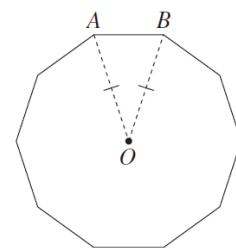
COMMON QUESTION: NESA 2020 Mathematics Standard 2 and Advanced HSC Examinations

MA SP 19 MS 2	12 Band 2-4 17	The diagram shows a triangle with sides of length *x* cm, 11 cm and 13 cm and an angle of 80°. Use the cosine rule to calculate the value of *x*, correct to two significant figures.		3	Solution
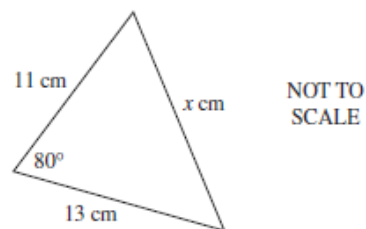
NESA Mathematics Advanced Sample Examination Paper (2020)
NESA 2019 Mathematics Standard 2 HSC Examination

| MA SP | 21 Band 3-5 | The diagram shows the distances of four towns A, B, C and D from point O.
The true bearings of towns A, B and D from point O are also shown.
The area of the acute-angled triangle BOC is 198 cm².
Calculate the true bearing of town C from point O, correct to the nearest degree. | | 3 | Solution |
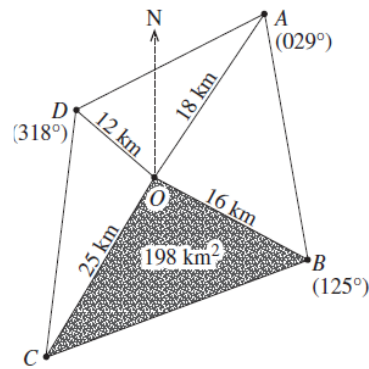
NESA Mathematics Advanced Sample Examination Paper (2020)

| MA SQ 2019 | 12 Band 2-5 | The diagram shows the three towns X, Y and Z. Town Z is due east of Town X. The bearing of Town Y from Town X is N39°E and the bearing of Town Z from Town Y is S51°E. The distance between Town X and Town Y is 1330 km.
A plane flies between the three towns. | | | Solution |
		(a) Mark the given information on the diagram and explain why $\angle XYZ$ is 90°.		2	
		(b) Find the distance between Town X and Town Z to the nearest kilometre.		2	
		(c) The plane is going to fly from Town Y to Town X, stopping at Town Z on the way. Leaving Town Y, the pilot incorrectly sets the bearing of Town Z to S50°E. The pilot flies for 1650 km before realising the mistake, then changes course and flies directly to Town X without going to Town Z. Which is closer to Town X: Town Z or the point where the pilot changes course? Justify your answer.		3	
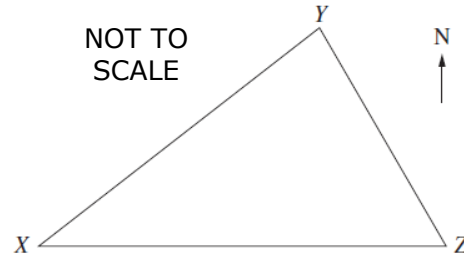
NESA Mathematics Advanced Sample examination materials (2019)

| TG | 1 | In the diagram, OAB is a sector of the circle with centre O and radius 6 cm, where $\angle AOB = 30°$. Determine the exact value of the area of the triangle OAB. | | | Solution |
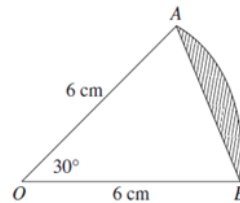
NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

| TG | 2 | Find the value of x in the following diagram. | | | Solution |
| 17 M | 13 a | | | | |

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions
NESA 2017 Mathematics HSC Examination

TG	**3**	Chris leaves island *A* in a boat and sails 142 km on a bearing of 078º to island *B*.	**Solution**
14	**13**	Chris then sails on a bearing of 191º for	
M	**d**	220 km to island *C*, as shown in the diagram.	

(a) Show that the distance from island *C* to island *A* is approximately 210 km.
(b) Chris wants to sail from island *C* directly to island *A*. On what bearing should Chris sail? Give your answer correct to the nearest degree.

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions
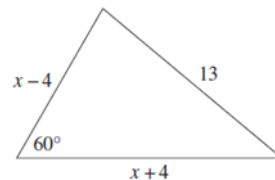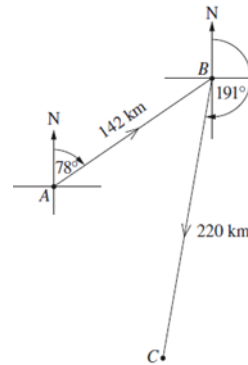NESA 2014 Mathematics HSC Examination

|			
TG	**4**	Determine the possible dimensions for triangle *ABC* given *AB* = 5.4 cm, $\angle BAC = 32º$ and *BC* = 3 cm.	**Solution**

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

TG	**5**	A person walks 2000 metres due north along a road from point *A* to point *B*. The point *A* is due east of a mountain *OM*, where *M* is the top of the mountain. The point *O* is directly below point *M* and is on the same horizontal plane as the road. The height of the mountain above point *O* is *h* metres. From point *A*, the angle of elevation to the top of the mountain is 15º. From point *B*, the angle of elevation to the top of the mountain is 13º. Determine the height of the mountain	**Solution**
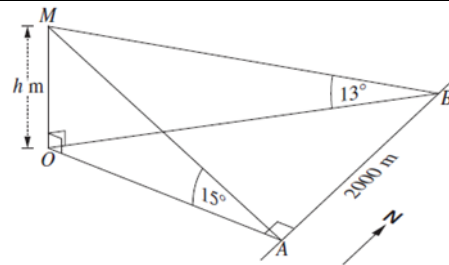
NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

TG	**6**	The Eiffel Tower is located in Paris, a city built on a flat floodplain. Three tourists *A*, *B* and *C* are observing the Eiffel Tower from the ground. *A* is due north of the tower, *C* is due east of the tower, and *B* is on the line-of-sight from *A* to *C* and between them. The angles of elevation to the top of the Eiffel Tower from *A*, *B* and *C* are 26º, 28º and 30º, respectively. Determine the bearing of *B* from the Eiffel Tower.	**Solution**

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

MS	**ME**	Which of the following expresses S20ºW as a true bearing?	**1**	**Solution**
SQ	**4**	A. 020º B. 070º C. 160º D. 200º		

NESA Mathematics Standard 2 Sample examination materials

MS	**ME**	Abbey walks 2 km due west from home to a coffee shop.	**1**	**Solution**
SQ	**7**	She then walks on a bearing of 148º to school, which is due south of her home. How far south, to the nearest 0.1 km, is Abbey from home?		

A. 1.1 km   B. 1.2 km   C. 3.2 km   D. 3.8 km

NESA Mathematics Standard 2 Sample examination materials

MS	**ME**	Paul travels from *A* to *B* on a bearing of 150º. He then turns and walks to *C* on a bearing of 055º. What is the size of $\angle ABC$?	**1**	**Solution**
SQ	**8**	A. 85º B. 90º C. 95º D. 115º		
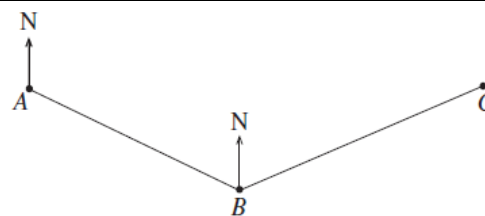
NOT TO SCALE
NESA Mathematics Standard 2 Sample examination materials

| MS SQ | ME 9 | The angle of depression from a window to a car on the ground is 40°. The car is 50 metres from the base of the building. How high above the ground is the window, correct to the nearest metre? A. 32 m B. 38 m C. 42 m D. 48 m | | 1 | Solution |

NOT TO SCALE

NESA Mathematics Standard 2 Sample examination materials

| MS SQ | ME 24 | The diagram shows the radial survey of a piece of land. (a) B is south west of O. What is the true bearing of C from O? (b) What is the area of angle of △AOB, to the nearest m²? | | 2 3 | Solution |

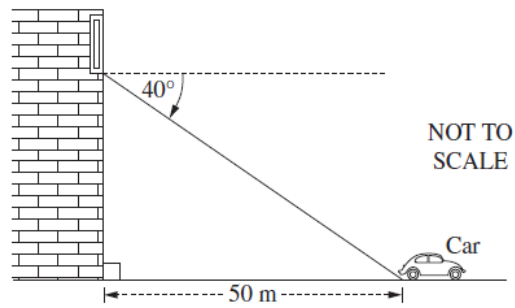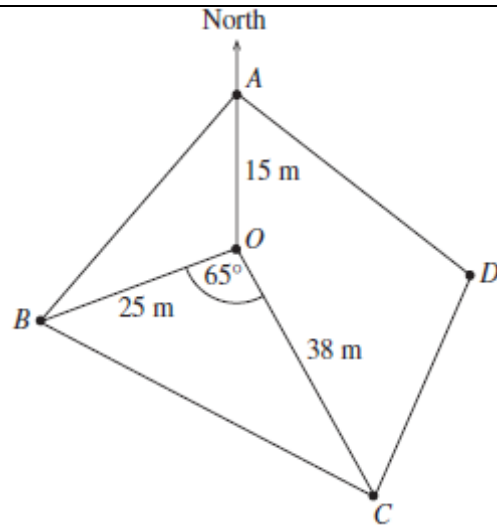NOT TO SCALE

NESA Mathematics Standard 2 Sample examination materials

| MS SQ | ME 25 | Lisa owns a piece of land as shown in the diagram. The length of BC is 230 metres. The size of angle BCA is 87° and of angle BAC is 47°. Lisa wants to build a fence along AC. Fencing can be purchased in metre lengths at a cost of $65 per metre. Calculate the cost of the fencing required. | | 4 | Solution |
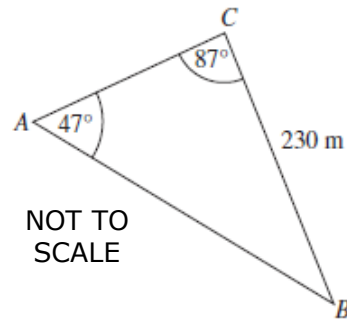
NOT TO SCALE

NESA Mathematics Standard 2 Sample examination materials

| MS SQ | ME 26 | Find the area of triangle PQR, correct to the nearest square metre. | | 4 | Solution |
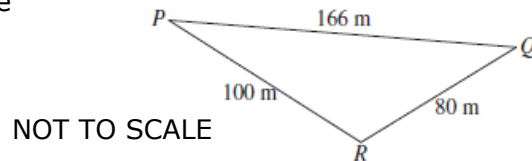
NOT TO SCALE

NESA Mathematics Standard 2 Sample examination materials

| MS SQ | ME 27 | The diagram shows triangle XYZ. The area of the triangle 43 m² and ∠YXZ is acute. What is the size of ∠YXZ, to the nearest degree? | | 3 | Solution |

NOT TO SCALE
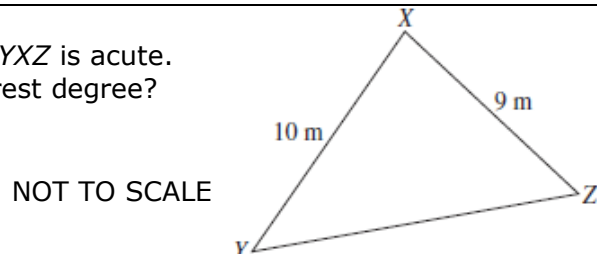
NESA Mathematics Standard 2 Sample examination materials

| MS SQ | ME 28 | The scale diagram shows the aerial view of a block of land bounded on one side by a road. The length of the block, *AB*, is known to be 45 metres. Calculate the approximate area of the block of land, using three applications of the trapezoidal rule.

[A note to students from *projectmaths*: Use a ruler to measure *AB* as 4.5 cm] | DIAGRAM TO SCALE

A

Road

B | 3 | Solution |

NESA Mathematics Standard 2 Sample examination materials

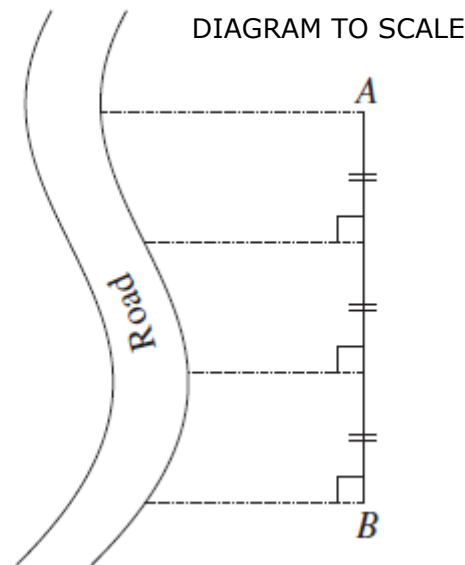| 19 M | 11 a | Using the sine rule, find the value of *x* correct to one decimal place. | 110°   *x*
40°   8 | 2 | Solution |
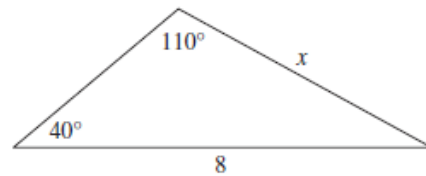
NESA 2019 Mathematics HSC Examination

| 19 M | 14 c | The regular hexagon *ABCDEF* has sides of length 1. The diagonal *AE* and the side *CD* are produced to meet at the point *X*.
Copy or trace the diagram into your writing booklet. Find the exact length of the line segment *EX*, justifying your answer. | *A F B E C D X* | 3 | Solution |
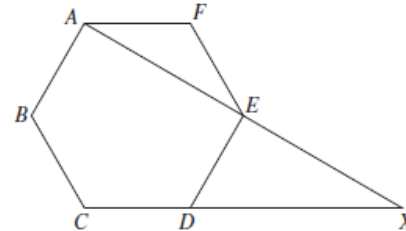
NESA 2019 Mathematics HSC Examination

| 19 MS 2 | 4 | Which compass bearing is the same as a true bearing of 110°?
A. S20°E   B. S20°W   C. S70°E   D. S70°W | 1 | Solution |

NESA 2019 Mathematics Standard 2 HSC Examination

| 19 MS 2 | 12 | An owl is 7 metres above ground level, in a tree. The owl sees a mouse on the ground at an angle of depression of 32°. How far must the owl fly in a straight line to catch the mouse, assuming the mouse does not move?
A. 3.7 m   B. 5.9 m   C. 8.3 m   D. 13.2 m | 1 | Solution |

NESA 2019 Mathematics Standard 2 HSC Examination

| 19 MS 2 | 22 | Two right-angled triangles, *ABC* and *ADC*, are shown.

Calculate the size of angle θ, correct to the nearest minute. | *B*
4.9 cm   NOT TO SCALE
A θ
2.5 cm
D 6 cm *C* | 3 | Solution |
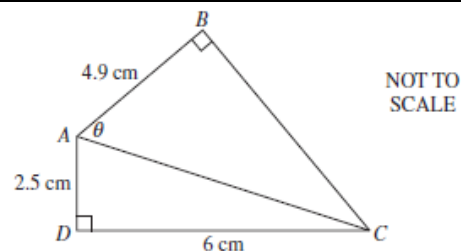
NESA 2019 Mathematics Standard 2 HSC Examination

19
MS
2

35 A compass radial survey shows the positions of four towns A, B, C and D relative to the point O.

The area of the triangle BOC is 198 km².

Calculate the bearing of town C from point O, correct to the nearest degree.
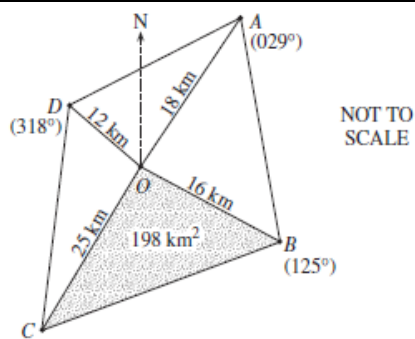
3

NESA 2019 Mathematics Standard 2 HSC Examination

*project*maths

Year 11: Trigonometric Functions
T1.2 Radians

Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017

Students:

- understand the unit circle definition of $\sin\theta$, $\cos\theta$ and $\tan\theta$ and periodicity using degrees (ACMMM029)
    - sketch the trigonometric functions in degrees for $0° \leq x \leq 360°$
- define and use radian measure and understand its relationship with degree measure (ACMMM032)
    - convert between the two measures, using the fact that $360° = 2\pi$ radians
    - recognise and use the exact values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ in both degrees and radians for integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$ (ACMMM035)
- understand the unit circle definition of $\sin\theta$, $\cos\theta$ and $\tan\theta$ and periodicity using radians (ACMMM034)
- solve problems involving trigonometric ratios of angles of any magnitude in both degrees and radians
- recognise the graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$ and sketch on extended domains in degrees and radians (ACMMM036)
- derive the formula for arc length, $l = r\theta$ and for the area of a sector of a circle, $A = \frac{1}{2}r^2\theta$
- solve problems involving sector areas, arc lengths and combinations of either areas or lengths

Reference Sheet

TG	1	Solve, if $0 \leq x \leq 2\pi$, (a) $\sqrt{3}\sin x = \cos x$ (b) $2\cos 2x = 0$ NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions	Solution
TG	2	Find the exact values of: (a) $\cos\frac{4}{3}\pi$ (b) $\sin\frac{5}{6}\pi$ (c) $\tan(-45°)$ NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions	Solution
TG 95 M	3 1c	Convert $\frac{3\pi}{5}$ radians to degrees. NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions NESA 1995 Mathematics HSC Examination	Solution
TG	4	Find the exact value of $\sin\left(\frac{\pi}{4}\right) + \sin\left(\frac{2\pi}{3}\right)$. NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions	Solution
TG	5	Solve $2\sin x = 1$ for $-2\pi \leq x \leq 2\pi$. NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions	Solution
TG	6	Find the perimeter and the area of the segment cut off by a chord PQ of length 8 cm in a circle centre O and radius 6 cm. Give your answers correct to 3 significant figures. NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions	Solution

TG	**7**	A chord of a circle which subtends an angle of θ at the centre of the circle cuts off a segment equal in area to $\frac{1}{3}$ of the area of the whole circle.

(a) Show that $\theta - \sin\theta = \frac{2\pi}{3}$.

(b) Verify that $\theta = 2.61$ radians, correct to 2 decimal places.

Solution

19 M	**13 b**	The diagram shows a circle with centre O and radius 20 cm. The points A and B lie on the circle such that $\angle AOB = 70°$. Find the perimeter of the shaded segment, giving your answer correct to one decimal place

NOT TO SCALE

3   **Solution**

projectmaths

Year 11: Statistical Analysis
S1.1 Probability and Venn Diagrams

Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017

Students:

- understand and use the concepts and language associated with theoretical probability, relative frequency and the probability scale 📎☞
- solve problems involving simulations or trials of experiments in a variety of contexts **AAM** 📎🖥
  - identify factors that could complicate the simulation of real-world events (ACMEM153)
  - use relative frequencies obtained from data as point estimates of probabilities (ACMMM055)
- use arrays and tree diagrams to determine the outcomes and probabilities for multi-stage experiments (ACMEM156) **AAM** 📎
- use Venn diagrams, set language and notation for events, including $\bar{A}$ (or A' or A^c) for the complement of an event A, $A \cap B$ for 'A and B', the intersection of events A and B, and $A \cup B$ for 'A or B', the union of events A and B, and recognise mutually exclusive events (ACMMM050) **AAM**
  - use everyday occurrences to illustrate set descriptions and representations of events and set operations (ACMMM051)
- establish and use the rules: $P(\bar{A}) = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (ACMMM054) **AAM** 📎
- understand the notion of conditional probability and recognise and use language that indicates conditionality (ACMMM056)
- use the notation $P(A|B)$ and the formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$, $P(B) \neq 0$ for conditional probability (ACMMM057) **AAM**
- understand the notion of independence of an event A from an event B, as defined by $P(A|B) = P(A)$ (ACMMM058)
- use the multiplication law $P(A \cap B) = P(A)P(B)$ for independent events A and B and recognise the symmetry of independence in simple probability situations (ACMMM059)

S T A N D A R D 2

S T -

Reference Sheet

---

23 MA	2

A game involves throwing a die and spinning a spinner.

The sum of the two numbers obtained is the score. The table of scores below is partially completed.

What is the probability of getting a score of 7 or more?

A. $\dfrac{1}{6}$ B. $\dfrac{1}{4}$

C. $\dfrac{5}{18}$ D. $\dfrac{5}{12}$

1

		SPINNER			
		1	2	3	4
DIE	1	2	3	4	
	2	3	4	5	
	3		5	6	
	4			7	
	5				
	6				

COMMON QUESTION: NESA 2023 Mathematics Standard 2 and Advanced HSC Examinations

23
MA

31   Four Year 12 students want to organise a graduation party. All four students have the same probability, $P(F)$, of being available next Friday. All four students have the same probability, $P(S)$, of being available next Saturday.

It is given that $P(F) = \dfrac{3}{10}$, $P(S|F) = \dfrac{1}{3}$, and $P(F|S) = \dfrac{1}{8}$.

Kim is one of the four students.

(a) Is Kim's availability next Friday independent from his availability next Saturday? Justify your answer.   **1**

(b) Show that the probability that Kim is available next Saturday is $\dfrac{4}{5}$.   **2**

(c) What is the probability that at least one of the four students is NOT available next Saturday?   **2**

NESA 2023 Mathematics Advanced HSC Examination

22
MA

9   Liam is playing two games. He is equally likely to win each game.   **1**
The probability that Liam will win at least one of the games is 80%.
Which of the following is closest to the probability that Liam will win both games?
A. 31%   B. 40%   C. 55%   D. 64%
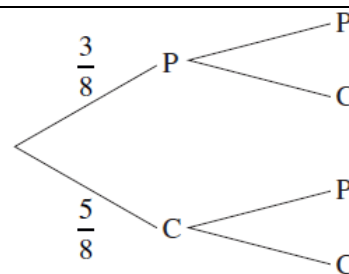
NESA 2022 Mathematics Advanced HSC Examination

22
MA

15   In a bag there are 3 six-sided dice.
Two of the dice have faces marked 1, 2, 3, 4, 5, 6.
The other is a special die with faces marked 1, 2, 3, 5, 5, 5.
One die is randomly selected and tossed.
(a) What is the probability that the die shows a 5?   **1**
(b) Given that the die shows a 5, what is the probability that it is the special die?   **1**

NESA 2022 Mathematics Advanced HSC Examination

21
MA

6   There are 8 chocolates in a box.   **1**
Three have peppermint centres (P) and five have caramel centres (C).
Kim randomly chooses a chocolate from the box and eats it. Sam then randomly chooses and eats one of the remaining chocolates.
A partially completed probability tree is shown.

What is the probability that Kim and Sam choose chocolates with different centres?

A. $\dfrac{15}{64}$ B. $\dfrac{15}{56}$ C. $\dfrac{15}{32}$ D. $\dfrac{15}{28}$

COMMON QUESTION: NESA 2021 Mathematics Standard 2 and Advanced HSC Examinations

20 **MA**	**14**	History and Geography are two subjects students may decide to study.	**Solution**

For a group of 40 students, the following is known.

- 7 students study neither History nor Geography
- 20 students study History
- 18 students study Geography

(a) A student is chosen at random. By using a Venn diagram, or otherwise, find the probability that the student studies both History and Geography. **2**

(b) A student is chosen at random. Given that the student studies Geography, what is the probability that the student does NOT study History? **1**

(c) Two different students are chosen at random, one after the other. What is the probability that the first student studies History and the second student does NOT study History? **2**

<div align="right">NESA 2020 Mathematics Advanced HSC Examination</div>

20 **MS** **2**	**15**	The top of a rectangular table is divided into 8 equal sections as shown.	**Solution**

1	2	3	4
5	6	7	8

A standard die with faces labelled 1 to 6 is rolled onto the table.

The die is equally likely to land in any of the 8 sections of the table. If the die does not land entirely in one section of the table, it is rolled again.

A score is calculated by multiplying the value shown on the top face of the die by the number shown in the section of the table where the die lands.

What is the probability of getting a score of 6?

A $\dfrac{1}{48}$ B $\dfrac{1}{12}$ C $\dfrac{1}{8}$ D $\dfrac{1}{6}$

<div align="right">NESA 2020 Mathematics Standard 2 HSC Examination</div>

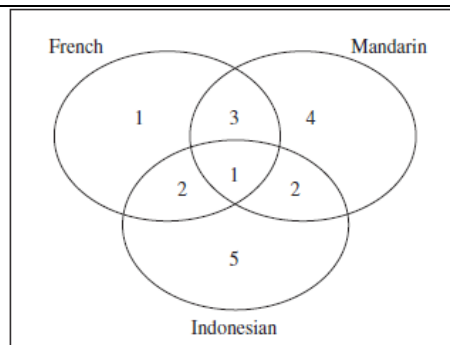|			
MA **SP**	**13** Band 2-4	A credit card requires a four-figure personal identification number (PIN) for purchases. The figures are chosen from the digits 0, 1, 2, 3, …, 9. Repetition is allowed and the PIN can start with any of the 10 digits. The credit card is lost and the finder tries to guess the PIN by entering four digits.	**Solution**

(a) What is the probability that the four digits entered are the correct PIN? **1**

(b) What is the probability that the finder will guess at least one digit in its correct order? **1**

<div align="right">NESA Mathematics Advanced Sample Examination Paper (2020)</div>
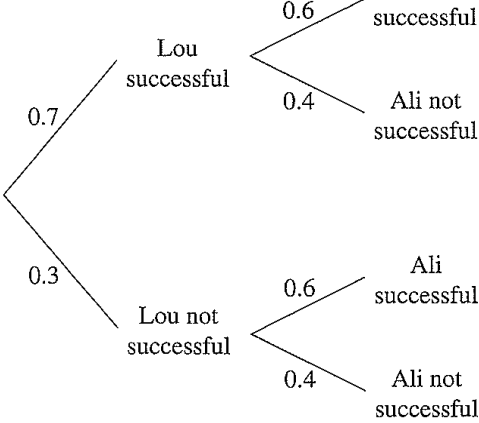
MA **SQ** **2019**	**5** Band 3-4	In a mixed language class, students study French, Mandarin and Indonesian. The number of students who study each language are shown in the Venn diagram. A student who studies Indonesian is selected at random. What is the probability that he/she also studies French?	**Solution**

A. $\dfrac{1}{6}$ B. $\dfrac{3}{10}$ C. $\dfrac{3}{5}$ D. $\dfrac{7}{10}$ **1**

<div align="right">NESA Mathematics Advanced Sample examination materials (2019)</div>

MA SQ 2019	16 Band 2-5	A survey found that in a large population approximately 20% of people are left-handed.
		(a) Three people are selected at random. **2**
		     Find the probability that at least one of them is left-handed.
		(b) What is the smallest number of people that would need to be selected to **3**
		     have a greater than 99% chance that at least one of them is left-handed?
		 NESA Mathematics Advanced Sample examination materials (2019)
			Solution

TG	1	Six girls' names and five boys' names are placed in a hat. Two names are drawn without replacement. What is the probability that a girl's and a boy's name are chosen? NESA Mathematics Advanced Year 11 Topic Guide: Statistical analysis	**Solution**
TG	2	In a raffle, 30 tickets are sold and there are two prizes. John buys five tickets. What is the probability that John wins at least one prize? NESA Mathematics Advanced Year 11 Topic Guide: Statistical analysis	**Solution**
TG	3	In Australia, approximately 9% of the population has the blood type O negative. If three people are chosen at random from the population, find the probability that: (a) none has O negative blood (b) at least one has O negative blood. NESA Mathematics Advanced Year 11 Topic Guide: Statistical analysis	**Solution**
TG	4	The manager of a team notices that the team has a probability of $\frac{2}{3}$ of winning the game if it is raining and if it is dry, the probability of the team winning is $\frac{1}{5}$. The probability that it will rain on a day when they play is $\frac{1}{4}$. (a) Find the probability that they will not win. (b) Given that the team has won a game, calculate the probability that it rained on the day of the match. NESA Mathematics Advanced Year 11 Topic Guide: Statistical analysis	**Solution**
TG 10 MG	5 20	Lou and Ali are on a fitness program for one month. The probability that Lou will finish the program successfully is 0.7 while the probability that Ali will finish successfully is 0.6. The probability tree diagram shows this information. What is the probability that only one of Lou and Ali will be successful? 0.7 — Lou successful — 0.6 Ali successful — 0.4 Ali not successful 0.3 — Lou not successful — 0.6 Ali successful — 0.4 Ali not successful NESA Mathematics Advanced Year 11 Topic Guide: Statistical analysis NESA 2010 General Mathematics HSC Examination	**Solution**
TG	6	A bag contains two red balls, one black ball, and one white ball. Andrew selects one ball from the bag and keeps it hidden. He then selects a second ball, also keeping it hidden. (a) Draw a tree diagram to show all the possible outcomes. (b) Find the probability that both the selected balls are red. (c) Find the probability that at least one of the selected balls is red. (d) Andrew drops one of the selected balls and we can see that it is red. What is the probability that the ball that is still hidden is also red? NESA Mathematics Advanced Year 11 Topic Guide: Statistical analysis	**Solution**

| 19 M | 6 | A game is played by tossing an ordinary 6-sided die and an ordinary coin at the same time. The game is won if the uppermost face of the die shows an even number or the uppermost face of the coin shows a tail (or both). What is the probability of winning this game? | 1 | **Solution** |

A. $\dfrac{1}{4}$ B. $\dfrac{1}{2}$ C. $\dfrac{3}{4}$ D. 1

| 19 M | 11 f | A bag contains 5 green beads and 7 purple beads. Two beads are selected at random, without replacement. What is the probability that the two beads are the same colour? | 2 | **Solution** |

| 19 M | 15 d | The probability that a person chosen at random has red hair is 0.02. | | **Solution** |

(i)   Two people are chosen at random.   **2**

     What is the probability that at least ONE has red hair?

(ii)   What is the smallest number of people that can be chosen at random so   **2**
that the probability that at least ONE has red hair is greater than 0.4

| 19 MS 2 | 20 | A roulette wheel has the numbers 0, 1, 2, …, 36 where each of the 37 numbers is equally likely to be spun.

If the wheel is spun 18 500 times, calculate the expected frequency of spinning the number 8. | 2 | **Solution** |

| 19 MS 2 | 25 | A bowl of fruit contains 17 apples of which 9 are red and 8 are green.
Dennis takes one apple at random and eats it. Margaret also takes an apple at random and eats it.
By drawing a probability tree diagram, or otherwise, find the probability that Dennis and Margaret eat apples of the same colour. | 3 | **Solution** |
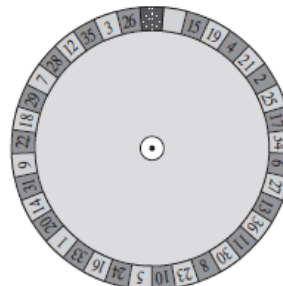
projectmaths

Year 12: Calculus
C3.2 Applications of the Derivative

Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017
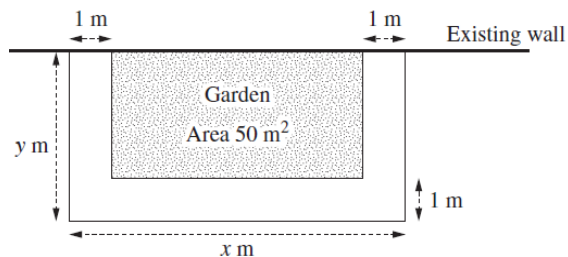
Students:

- use any of the functions covered in the scope of this syllabus and their derivatives to solve practical and abstract problems **AAM**
- use calculus to determine and verify the nature of stationary points, find local and global maxima and minima and points of inflection (horizontal or otherwise), examine behaviour of a function as $x \to \infty$ and $x \to -\infty$ and hence sketch the graph of the function (ACMMM095) ⚙️🖥️🎓
- solve optimisation problems for any of the functions covered in the scope of this syllabus, in a wide variety of contexts including displacement, velocity, acceleration, area, volume, business, finance and growth and decay **AAM** 🗡️⚙️🖥️🎓
  - define variables and construct functions to represent the relationships between variables related to contexts involving optimisation, sketching diagrams or completing diagrams if necessary
  - use calculus to establish the location of local and global maxima and minima, including checking endpoints of an interval if required
  - evaluate solutions and their reasonableness given the constraints of the domain and formulate appropriate conclusions to optimisation problems

Reference Sheet

---

23 MA | **24** | A gardener wants to build a rectangular garden of area 50 m² against an existing wall as shown in the diagram. A concrete path of width 1 metre is to be built around the other three sides of the garden.

Let x and y be the dimensions, in metres, of the outer rectangle as shown.

1 m | 1 m | Existing wall

Garden
Area 50 m²

y m

1 m

x m

Solution

(a) Show that $y = \dfrac{50}{x-2} + 1$.   **1**

(b) Find the value of x such that the area of the concrete path is a minimum.   **4**

Show that your answer gives a minimum area.

NESA 2023 Mathematics Advanced HSC Examination

---

23 MA | **30** | Let $f(x) = e^{-x \sin x}$.
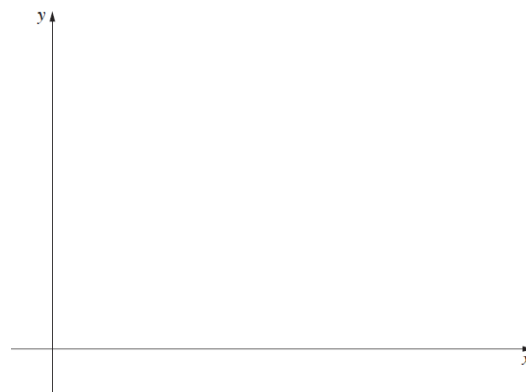
(a) Find the coordinates of the stationary points of $f(x)$ for $0 \le x \le 2\pi$.
You do NOT need to check the nature of the stationary points.   **3**

(b) Without using any further calculus, sketch the graph of $f(x)$ for $0 \le x \le 2\pi$, showing stationary points and intercepts.   **2**

y

x

Solution

NESA 2023 Mathematics Advanced HSC Examination

---

22 MA	20	A scientist is studying the growth of bacteria.	Solution

A scientist is studying the growth of bacteria.

The scientist models the number of bacteria, N, by the equation $N(t) = 200e^{0.013t}$, where t is the number of hours after starting the experiment.

(a) What is the initial number of bacteria in the experiment? **1**

(b) What is the number of bacteria 24 hours after starting the experiment? **1**

(c) What is the rate of increase in the number of bacteria 24 hours after starting the experiment? **2**

NESA 2022 Mathematics Advanced HSC Examination

---

22 MA   22   Find the global maximum and minimum values of $y = x^3 - 6x^2 + 8$, where $-1 \le x \le 7$.   **4**   Solution

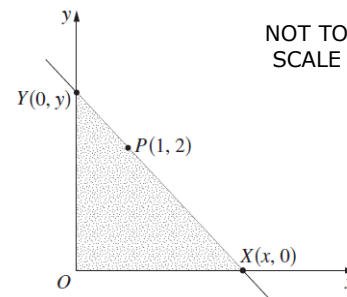*NESA 2022 Mathematics Advanced HSC Examination*

---

22 MA   27   Let $f(x) = xe^{-2x}$.

It is given that $f'(x) = e^{-2x} - 2xe^{-2x}$.

(a) Show that $f''(x) = 4(x - 1)e^{-2x}$. **2**

(b) Find any stationary points of $f(x)$ and determine their nature. **2**

(c) Sketch the curve $y = xe^{-2x}$, showing any stationary points, points of inflection and intercepts with the axes. **3**

NESA 2022 Mathematics Advanced HSC Examination

---

22 MA   31   A line passes through the point $P(1, 2)$ and meets the axes at $X(x, 0)$ and $Y(0, y)$, where $x > 1$.

(a) Show that $y = \dfrac{2x}{x - 1}$. **2**

(b) Find the minimum value of the area of triangle XOY. **4**

NOT TO SCALE

$Y(0, y)$

$P(1, 2)$

$X(x, 0)$

O

NESA 2022 Mathematics Advanced HSC Examination

---

21 MA   26   A particle is shot vertically upwards from a point 100 metres above ground level.

The position of the particle, y metres above the ground after t seconds, is given by

$y(t) = -5t^2 + 70t + 100$.

(a) Find the maximum height above ground level reached by the particle. **2**

(b) Find the velocity of the particle, in metres per second, immediately before it hits the ground, leaving your answer in the form $a\sqrt{b}$, where a and b are integers. **3**

NESA 2021 Mathematics Advanced HSC Examination

20 MA	21	Hot tea is poured into a cup.		

The temperature of tea can be modelled by $T = 25 + 70(1.5)^{-0.4t}$, where T is the temperature of the tea, in degrees Celsius, t minutes after it is poured.

(a) What is the temperature of the tea 4 minutes after it has been poured?   **1**
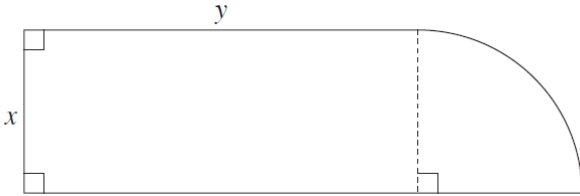
(b) At what rate is the tea cooling 4 minutes after it has been poured?   **2**

(c) How long after the tea is poured will it take for its temperature to reach 55   **3**

NESA 2020 Mathematics Advanced HSC Examination

---

20 MA 25   A landscape gardener wants to build a garden bed in the shape of a rectangle attached to a quarter-circle.

Let x and y be the dimensions of the rectangle in metres, as shown in the diagram.

The garden bed is required to have an area of 36 m^2 and to have a perimeter which is as small as possible. Let P metres be the perimeter of the garden bed.

(a) Show that $P = 2x + \dfrac{72}{x}$.   **3**

(b) Find the smallest possible perimeter of the garden bed, showing why this is the minimum perimeter.   **4**
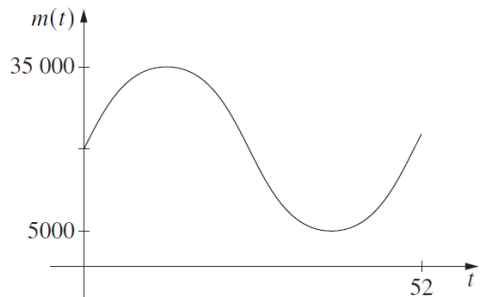
NESA 2020 Mathematics Advanced HSC Examination

---

20 MA 31   The population of mice on an isolated island can be modelled by the function

$m = a\sin\left(\dfrac{\pi}{26}t\right) + b$, where t is the time in

weeks and $0 \le t \le 52$. The population of mice reaches a maximum of 35 000 when $t = 13$ and a minimum of 5000 when $t = 39$.

The graph of $m(t)$ is shown.

(a) What are the values of a and b?   **2**

(b) On the same island, the population of cats can be modelled by the function   **3**

$c(t) = -80 \cos\left(\dfrac{\pi}{26}(t-10)\right) + 120$.

Consider the graph of $m(t)$ and the graph of $c(t)$.

Find the values of t, $0 \le t \le 52$, for which both populations are increasing.

(c) Find the rate of change of the mice population when the cat population reaches a maximum.   **2**

NESA 2020 Mathematics Advanced HSC Examination

MA SP 14 Band 2-5   A function is given by $f(x) = 18x^2 - x^4$.

(a) Find the stationary points and determine their nature.   **4**

(b) Sketch the curve, labelling the stationary points and axis intercepts.   **2**

NESA Mathematics Advanced Sample Examination Paper (2020)

| MA SP | 32 Band 2-5 | A farmer wishes to make a rectangular enclosure of area 720 m². She uses an existing straight boundary as one side of the enclosure. She uses wire fencing for the remaining three sides and also to divide the enclosure into four equal rectangular areas of width x m as shown. | Existing boundary | 3 | **Solution** |

| 16 M | 14 C |
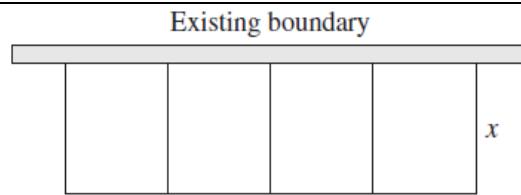
The total length, ℓ m, of the wire fencing is given by $\ell = 5x + \dfrac{720}{x}$.

(Do NOT prove this.)
Find the minimum length of wire fencing required, showing why this is the minimum length.

<div align="right">

NESA Mathematics Advanced Sample HSC Examination Paper (2020)
NESA 2016 Mathematics HSC Examination
</div>

| TG | 1 | By finding stationary points and their nature and intercepts on axes, sketch the curve $y = e^{-x}\sin 2x$ over the domain $0 \le x \le \pi$. | **Solution** |

<div align="right">NESA Mathematics Advanced Year 12 Topic Guide: Calculus</div>

| TG | 2 | Consider the curve $y = \dfrac{1}{4}x^4 - x^3$. | **Solution** |

(a) Find any stationary points and determine their nature.
(b) Find any points of inflection.
(c) Sketch the curve for $-1.5 \le x \le 4.5$, indicating where the curve crosses the x-axis.
(d) For what values of x is the curve concave down?
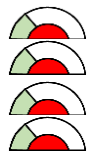
<div align="right">NESA Mathematics Advanced Year 12 Topic Guide: Calculus</div>

| TG | 3 | A box without a lid is made by cutting out four equal squares from the corners of a sheet of heavy card, then folding up the sides.
If the card has dimensions 20 cm by 10 cm, what are the dimensions of the box with largest volume that can be constructed in this way? | **Solution** |

<div align="right">NESA Mathematics Advanced Year 12 Topic Guide: Calculus</div>

| TG | 4 | A right circular cone is inscribed in a sphere of radius a, centred at O. The distance from the base of the cone to the top of the sphere is x and the radius of the base is r, as shown in the diagram. | **Solution** |

| 06 M | 9c |
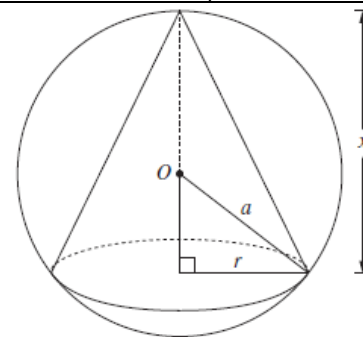
(a) Show that the volume, V, of the cone is given by $V = \dfrac{1}{3}\pi(2ax^2 - x^3)$.

(b) Find the value of x for which the volume of the cone is a maximum.
Give reasons why this value of x gives the maximum volume.

<div align="right">

NESA Mathematics Advanced Year 12 Topic Guide: Calculus
NESA 2006 Mathematics HSC Examination
</div>

| 19 M | 10 | A particle is moving along a straight line with displacement x at time t. | 1 | **Solution** |

The particle is stationary when $t = 11$ and when $t = 13$.
Which of the following MUST be true in this case?

A. The particle changes direction at some time between $t = 11$ and $t = 13$.
B. The displacement function of the particle has a stationary point at some time between $t = 11$ and $t = 13$.
C. The acceleration of the particle is 0 at some time between $t = 11$ and $t = 13$.
D. The acceleration function of the particle has a stationary point at some time between $t = 11$ and $t = 13$.

<div align="right">NESA 2019 Mathematics HSC Examination</div>

| 19 M | 12 c | The number of leaves, $L(t)$, on a tree t days after the start of autumn can be modelled by $L(t) = 200\,000e^{-0.14t}$. | | |

(i)   What is the number of leaves on the tree when $t = 31$?   **1**

(ii) What is the rate of change of the number of leaves on the tree when $t = 31$?   **2**

(iii) For what value of t are there 100 leaves on the tree?   **2**

| 19 M | 14 b | The derivative of a function $y = f(x)$ is given by $f'(x) = 3x^2 + 2x - 1$. | | |

(i)   Find the x-values of the two stationary points of $y = f(x)$, and determine the nature of the stationary points.   **2**

(ii) The curve passes through the point (0, 4). Find an expression for $f(x)$.   **2**

(iii) Hence sketch the curve, clearly indicating the stationary points.   **2**

(iv) For what values of x is the curve concave down?   **1**

| 19 M | 15 c | The entry points, R and Q, to a national park can be reached via two straight access roads. The access roads meet the national park boundaries at right angles. The corner, P, of the national park is 8 km from R and 1 km from Q. The boundaries of the national park form a right angle at P. A new straight road is to be built joining these roads and passing through P. Points A and B on the access roads are | | |

to be chosen to minimise the distance, D km, from A to B along the new road. Let the distance QA be x km.
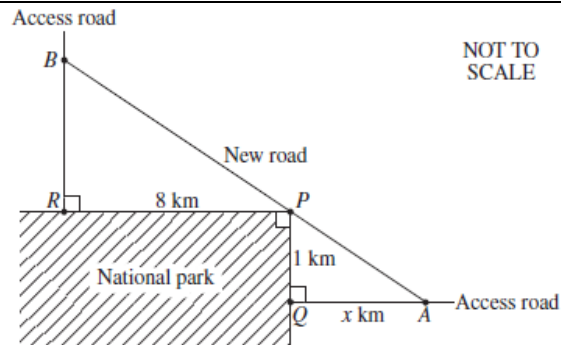
(i) Show that $D^2 = (x + 8)^2 + (\dfrac{8}{x} + 1)^2$ **3**

(ii) Show that $x = 2$ gives the minimum value of D^2.   **3**

Year 12: Statistical Analysis
S3.1 Continuous Random Variables

Back

Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017

Students:

- use relative frequencies and histograms obtained from data to estimate probabilities associated with a continuous random variable (ACMMM164)
- understand and use the concepts of a probability density function of a continuous random variable **AAM**
    - know the two properties of a probability density function: $f(x) \geq 0$ for all real x and $\int_{-\infty}^{\infty} f(x)dx = 1$
    - define the probability as the area under the graph of the probability density function using the notation $P(X \leq r) = \int_{a}^{r} f(x)dx$, where $f(x)$ is the probability density function defined on $[a, b]$
    - examine simple types of continuous random variables and use them in appropriate contexts ⚙
    - explore properties of a continuous random variable that is uniformly distributed
    - find the mode from a given probability density function
- obtain and analyse a cumulative distribution function with respect to a given probability density function
    - understand the meaning of a cumulative distribution function with respect to a given probability density function
    - use a cumulative distribution function to calculate the median and other percentiles

Reference Sheet

---

23 MA **29** A continuous random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} 12x^2(1-x), & \text{for } 0 \leq x \leq 1 \\ 0, & \text{for all other values of x} \end{cases}$$

Solution

(a) Find the mode of X. **2**

(b) Find the cumulative distribution function for the given probability density function. **2**

(c) Without calculating the median, show that the mode is greater than the median. **2**

NESA 2023 Mathematics Advanced HSC Examination

---

22 MA **7** Consider the following graph of a probability density function $f(x)$.
What is the value of the mode?

A. $\dfrac{1}{\pi}$ B. $\dfrac{3}{2\pi}$

C. $\dfrac{\pi}{4}$ D. π
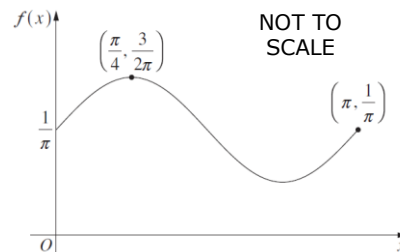
$f(x)$

NOT TO SCALE

$\left(\dfrac{\pi}{4}, \dfrac{3}{2\pi}\right)$

$\dfrac{1}{\pi}$

$\left(\pi, \dfrac{1}{\pi}\right)$

O x

1 **Solution**

NESA 2022 Mathematics Advanced HSC Examination

22   **30**   A continuous random variable X has cumulative
MA   distribution function given by:

         (a) Show that $k = 3$.
         (b) Given that $P(X < c) = 2P(X > c)$, find the
             exact value of c.

$$F(x) = \begin{cases} 1 & x > e^3 \\ \dfrac{1}{k}\ln x & 1 \le x \le e^3 \\ 0 & x < 1 \end{cases}$$

1
2

21   **30**   The number of hours for which light bulbs will work before failing can be modelled by the **2**
MA   random variable X with cumulative distribution function.

$$F(x) = \begin{cases} 1 - e^{-0.01x}, & x \ge 0 \\ 0, & x < 0 \end{cases}$$

Jane sells light bulbs and promises that they will work for longer than exactly 99% of all
light bulbs. Find how long, according to Jane's promise, a light bulb bought from her
should work. Give your answer in hours, rounded to two decimal place

21   **33**   People are given a maximum of six hours to complete a puzzle. The time spent on the
MA   puzzle, in hours, can be modelled using the continuous random variable X which has
     probability density function

$$f(x) = \begin{cases} \dfrac{Ax}{x^2 + 4}, & \text{for } 0 \le x \le 6, (\text{where } A > 0) \\ 0, & \text{for all other values of } x \end{cases}$$

The graph of the probability
density function is shown below.
The graph has a local maximum.
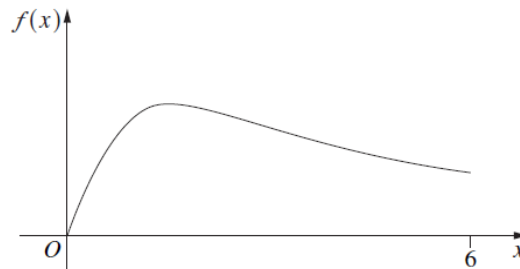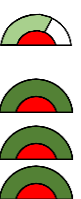
(a) Show that $A = \dfrac{2}{\ln 10}$.   **2**

(b) Show that the mode of X is two hours.   **2**

(c) Show that $P(X < 2) = \log_{10} 2$.   **2**

(d) The Intelligence Quotient (IQ) scores of people are normally distributed with a   **2**
     mean of 100 and standard deviation of 15. It has been observed that the puzzle is
generally completed more quickly by people with a high IQ. It is known that   80% of
people with an IQ greater than 130 can complete the puzzle in less than   two hours.
A person chosen at random can complete the puzzle in less than two   hours.

     What is the probability that this person has an IQ greater than 130? Give your
     answer correct to three decimal places.

20 MA	**23**	A continuous random variable, X, has the following probability density functions.	**Solution**

$$f(x) = \begin{cases} \sin x & \text{for } 0 \le x \le k \\ 0 & \text{for all other values of } k \end{cases}$$

(a) Find the value of k. **2**

(b) Find $P(X \le 1)$. Give your answer correct to four decimal places. **2**

<div align="right">NESA 2020 Mathematics Advanced HSC Examination</div>

MA SP **7** **Band 4-5**

The diagram shows the graph of a continuous probability density function.
Which of the following is the mode?

A. 0.07   B. 0.6
C. 0.8   D. 3
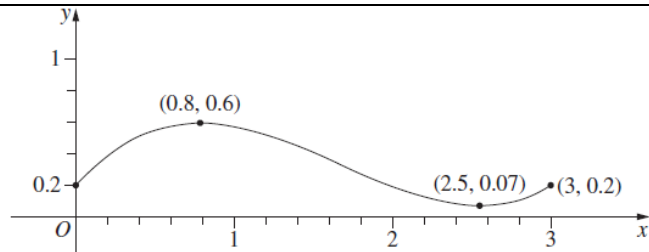
1 **Solution**

<div align="right">NESA Mathematics Advanced Sample HSC Examination Paper (2020)</div>

MA SP **31** **Band 3-6**

A bid made at an auction for a real estate property, in millions of dollars, can be modelled by the random variable X with the probability density function

$$f(x) = \begin{cases} k(16 - x^2) & 1 \le x \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Show that the value of k is $\dfrac{1}{27}$. **2**

(b) Find the cumulative distribution function. **2**

(c) Find the probability that a bid of more than 3 million dollars will be made. **1**

Solution

<div align="right">NESA Mathematics Advanced Sample Examination Paper (2020)</div>

MA SQ 2019 **13** **Band 2-5**

A continuous random variable X has a probability density function f given by

$$f(x) = \begin{cases} Ax + B & 1 \le x \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

where A and B are constants.
The median of X is 2.
Find the values of A and B.

4 **Solution**

<div align="right">NESA Mathematics Advanced Sample examination materials (2019)</div>

TG **1**

Let X be a continuous random variable whose probability density function is $f(x) = 3x^2$ for $0 < x < 1$.

(a) What is the probability that X falls between $\dfrac{1}{2}$ and 1?

(b) Show that $P(X = \dfrac{1}{2}) = 0$.

Solution

<div align="right">NESA Mathematics Advanced Year 12 Topic Guide: Statistical Analysis</div>

TG **2**

Let X be a continuous random variable whose probability density function is $f(x) = \dfrac{x^3}{4}$

for an interval $0 < x < c$.
What is the value of the constant c that makes $f(x)$ a valid probability density function?

Solution

<div align="right">NESA Mathematics Advanced Year 12 Topic Guide: Statistical Analysis</div>

TG **3**

Let X be a continuous random variable with the following probability density function:

$f(x) = \dfrac{1}{2}$ for $0 < x < 2$.

What are the lower quartile, median and upper quartile of X?

Solution

<div align="right">NESA Mathematics Advanced Year 12 Topic Guide: Statistical Analysis</div>

2023 HSC Paper

| 23 MA | 1 | The number of bees leaving a hive was observed and recorded over 14 days at different times of the day.

Which Pearson's correlation coefficient best describes the observations?

A. −0.8   B. −0.2

C. 0.2   D. 0.8 | | 1 | Solution |

<p align="center" style="color:red">COMMON QUESTION: NESA 2023 Mathematics Standard 2 and Advanced HSC Examinations</p>

| 23 MA | 2 | A game involves throwing a die and spinning a spinner.

The sum of the two numbers obtained is the score. The table of scores below is partially completed.

What is the probability of getting a score of 7 or more?

A. $\dfrac{1}{6}$ B. $\dfrac{1}{4}$

C. $\dfrac{5}{18}$ D. $\dfrac{5}{12}$ | | 1 | Solution |

The table of scores:

		SPINNER			
		1	2	3	4
DIE	1	2	3	4	
	2	3	4	5	
	3		5	6	
	4			7	
	5				
	6				

<p align="center" style="color:red">COMMON QUESTION: NESA 2023 Mathematics Standard 2 and Advanced HSC Examinations</p>

| 23 MA | 3 | What is the domain of $f(x) = \dfrac{1}{\sqrt{1-x}}$?

A. $x < 1$ B. $x \le 1$ C. $x > 1$ D. $x \ge 1$ | 1 | Solution |

<p align="right">NESA 2023 Mathematics Advanced HSC Examination</p>

| 23 MA | 4 | The graph of a polynomial is shown.

Which row of the table is correct for this polynomial? | | 1 | Solution |

	Equation	Value of b	Value of c
A.	$y = -(x - b)(x - c)^2$	$b > 0$	$c < 0$
B.	$y = -(x - b)(x - c)^2$	$b < 0$	$c > 0$
C.	$y = -x(x - b)(x - c)$	$b > 0$	$c < 0$
D.	$y = -x(x - b)(x - c)$	$b < 0$	$c > 0$

<p align="center">NESA 2023 Mathematics Advanced HSC Examination</p>

23 MA | **5** | The diagram shows the graph $y = f(x)$, where $f(x)$ is an odd function.

The shaded area is 1 square unit.

The number a, where $a > 1$, is chosen so that $\int_0^a f(x)\, dx = 0$.

NOT TO SCALE

What is the value of $\int_{-a}^1 f(x)\, dx$?

A. −1   B. 0

C. 1   D. 3

1   Solution

23 MA | **6** | The following table gives the signs of the first and second derivatives of a function $y = f(x)$ for different values of x.

x	−2	0	2
$f'(x)$	+	0	+
$f''(x)$	−	0	+

Which of the following is a possible sketch of $y = f(x)$?

1   Solution

23 MA | **7** | It is given that $y = f(g(x))$, where $f(1) = 3$, $f'(1) = -4$, $g(5) = 1$ and $g'(5) = 2$.

What is the value of y' at $x = 5$?

A. −8   B. −4   C. 3   D. 6

1   Solution

23 MA | **8** | What is the solution of the equation $\log_a x^3 = b$, where a and b are positive constants?

A. $x = b^{\frac{a}{3}}$ B. $x = a^{\frac{b}{3}}$ C. $x = \dfrac{b^a}{3}$ D. $x = \dfrac{a^b}{3}$

1   Solution

23 MA | **9** | Let $f(x)$ be any function with domain all real numbers.

Which of the following is an even function, regardless of the choice of $f(x)$?

A. $2f(x)$ B. $f(f(x))$ C. $f(-x)^2$ D. $f(x)f(-x)$
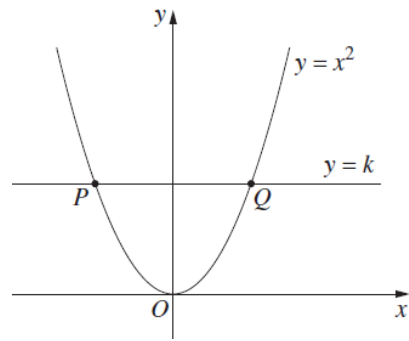
1   Solution

23 MA **10** The graph $y = x^2$ meets the line $y = k$ (where $k > 0$) at points P and Q as shown in the diagram.

The length of the interval PQ is L.

Let a be a positive number. The graph $y = \dfrac{x^2}{a^2}$ meets the line $y = k$ at points S and T.

What is the length of ST?

A. $\dfrac{L}{a}$ B. $\dfrac{L}{a^2}$

C. aL D. a^2L

1

NESA 2023 Mathematics Advanced HSC Examination

23 MA **11** The first three terms of an arithmetic sequence are 3, 7 and 11.

Find the 15th term.

2

NESA 2023 Mathematics Advanced HSC Examination

23 MA **12** The table shows the probability distribution of a discrete random variable.

x	0	1	2	3	4
$P(X = x)$	0	0.3	0.5	0.1	0.1

(a) Show that the expected value $E(X) = 2$.

(b) Calculate the standard deviation, correct to one decimal place.

1

2

NESA 2023 Mathematics Advanced HSC Examination

23 MA **13** Let $P(t)$ be a function such that $\dfrac{dP}{dt} = 3000e^{2t}$.

When $t = 0$, $P = 4000$.

Find an expression for $P(t)$.

2

NESA 2023 Mathematics Advanced HSC Examination

23 MA **14** Find the equation of the tangent to the curve $y = (2x + 1)^3$ at the point $(0, 1)$.

3

NESA 2023 Mathematics Advanced HSC Examination

23 MA   **15**   A table of future value interest factors for an annuity of $1 is shown.

Period \ Rate	1.5%	3%	4.5%	6%
5	5.152	5.309	5.471	5.637
10	10.703	11.464	12.288	13.181
20	23.124	26.870	31.371	36.786
40	54.268	75.401	107.030	154.762

(a) Micky wants to save $450 000 over the next 10 years.   **2**

If the interest rate is 6% per annum compounding annually, how much should Micky contribute each year? Give your answer to the nearest dollar.

(b) Instead, Micky decides to contribute $8535 every three months for 10 years to an annuity paying 6% per annum, compounding quarterly.   **3**

How much will Micky have at the end of 10 years?

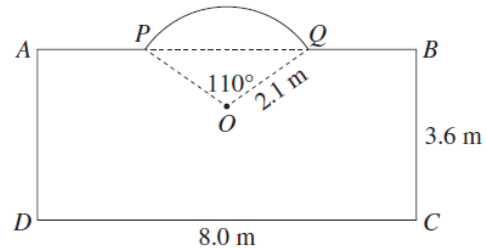COMMON QUESTION: NESA 2023 Mathematics Standard 2 and Advanced HSC Examinations

23 MA   **16**   The diagram shows a shape APQBCD. The shape consists of a rectangle ABCD with an arc PQ on side AB and with side lengths BC = 3.6 m and CD = 8.0 m.   **4**

The arc PQ is an arc of a circle with centre O and radius 2.1 m and $\angle POQ = 110°$.

What is the perimeter of the shape APQBCD? Give your answer correct to one decimal place.

NOT TO SCALE

COMMON QUESTION: NESA 2023 Mathematics Standard 2 and Advanced HSC Examinations

23 MA   **17**   Find $\int x\sqrt{x^2 + 1}\ dx$.   **2**

NESA 2023 Mathematics Advanced HSC Examination

Solution

23 MA **18**

A university uses gas to heat its buildings. Over a period of 10 weekdays during winter, the gas used each day was measured in megawatts (MW) and the average outside temperature each day was recorded in degrees Celsius (°C).

Using x as the average daily outside temperature and y as the total daily gas usage, the equation of the least-squares regression line was found.
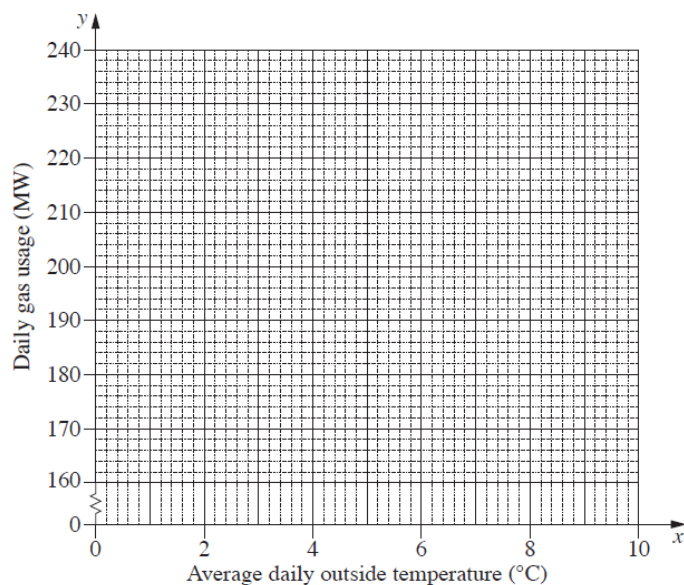
The equation of the regression line predicts that when the temperature is 0°C, the daily gas usage is 236 MW.

The ten temperatures measured were: 0°, 0°, 0°, 2°, 5°, 7°, 8°, 9°, 9°, 10°.

The total gas usage for the ten weekdays was 1840 MW.

In any bivariate dataset, the least-squares regression line passes through the point $(\bar{x}, \bar{y})$, where $\bar{x}$ is the sample mean of the x-values and $\bar{y}$ is the sample mean of the y-values.

(a) Using the information provided, plot the point $(\bar{x}, \bar{y})$ and the y-intercept of the least-squares regression line on the grid.   **3**

(b) What is the equation of the regression line?   **2**

(c) In the context of the dataset, identify ONE problem with using the regression line to predict gas usage when the average outside temperature is 23°C.   **1**
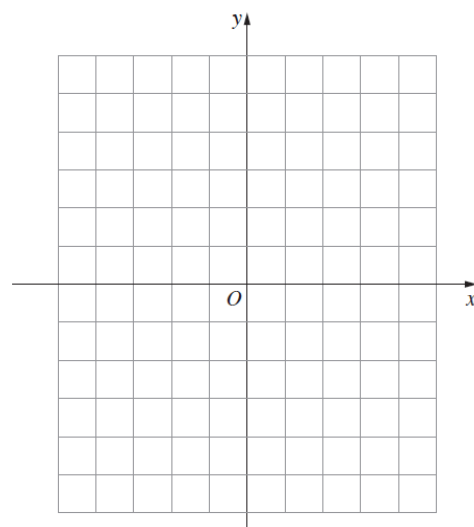

COMMON QUESTION: NESA 2023 Mathematics Standard 2 and Advanced HSC Examinations

23 MA **19**

(a) Sketch the graphs of the functions $f(x) = x - 1$ and $g(x) = (1 - x)(3 + x)$ showing the x-intercepts.   **2**

(b) Hence, or otherwise, solve the inequality $x - 1 < (1 - x)(3 + x)$.   **2**

NESA 2023 Mathematics Advanced HSC Examination

| 23 MA | 20 | Find all the values of θ, where $0° \le \theta \le 360°$, such that $\sin(\theta - 60°) = -\dfrac{\sqrt{3}}{2}$. | 3 | Solution |

NESA 2023 Mathematics Advanced HSC Examination

| 23 MA | 21 | The fourth term of a geometric sequence is 48. | 3 | Solution |

The eighth term of the same sequence is $\dfrac{3}{16}$.

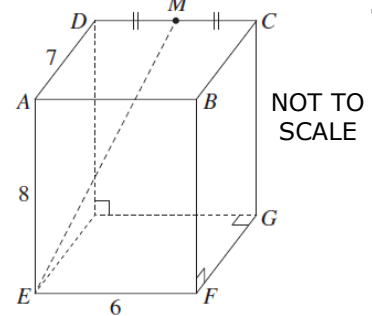Find the possible value(s) of the common ratio and the corresponding first term(s).

NESA 2023 Mathematics Advanced HSC Examination

| 23 MA | 22 | In the rectangular prism shown, AD = 7 cm, AE = 8 cm, EF = 6 cm. Point M is the midpoint of CD. | 3 | Solution |

Find $\angle AEM$, to the nearest degree.

NOT TO SCALE

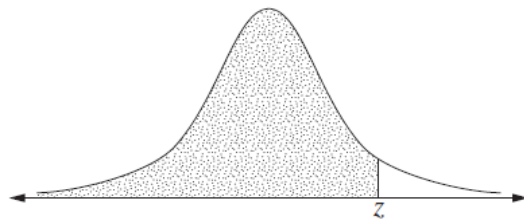NESA 2023 Mathematics Advanced HSC Examination

| 23 MA | 23 | A random variable is normally distributed with a mean of 0 and a standard deviation of 1. The table gives the probability that this random variable lies below z for some positive values of z. | 4 | Solution |

z	1.30	1.31	1.32	1.33	1.34	1.35	1.36	1.37	1.38	1.39
Probability	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177

The probability values given in the table are represented by the shaded area in the following diagram.

The weights of adult male koalas form a normal distribution with mean μ = 10.40 kg, and standard deviation σ = 1.15 kg.

In a group of 400 adult male koalas, how many would be expected to weigh more than 11.93 kg?

COMMON QUESTION: NESA 2023 Mathematics Standard 2 and Advanced HSC Examinations

| 23 MA | 24 | A gardener wants to build a rectangular garden of area 50 m² against an existing wall as shown in the diagram. A concrete path of width 1 metre is to be built around the other three sides of the garden. | | Solution |

A gardener wants to build a rectangular garden of area 50 m² against an existing wall as shown in the diagram. A concrete path of width 1 metre is to be built around the other three sides of the garden.
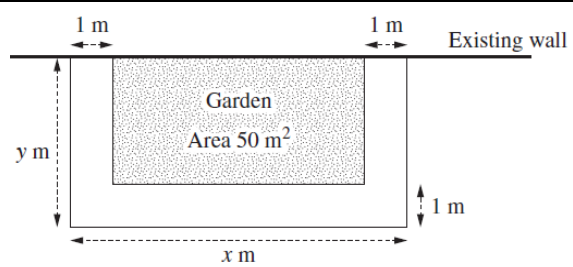
Let x and y be the dimensions, in metres, of the outer rectangle as shown.

(a) Show that $y = \dfrac{50}{x-2} + 1$. **1**

(b) Find the value of x such that the area of the concrete path is a minimum. **4**

   Show that your answer gives a minimum area.

NESA 2023 Mathematics Advanced HSC Examination

---

23 MA 25 On the first day of November, Jia deposits $10 000 into a new account which earns 0.4% interest per month, compounded monthly. At the end of each month, after the interest is added to the account, Jia intends to withdraw $M from the account.
Let A_n be the amount (in dollars) in Jia's account at the end of n months.

(a) Show that $A_2 = 10\,000(1.004)^2 - M(1.004) - M$ **1**

(b) Show that $A_n = (10\,000 - 250M)(1.004)^n + 250M$. **3**

(c) Jia wants to be able to make at least 100 withdrawals. **2**

   What is the largest value of M that will enable Jia to do this?

NESA 2023 Mathematics Advanced HSC Examination

---

23 MA 26 A camera films the motion of a swing in a park.
Let $x(t)$ be the horizontal distance, in metres, from the camera to the seat of the swing at t seconds.
The seat is released from rest at a horizontal distance of 11.2 m from the camera.

NOT TO SCALE

Camera

11.2 m
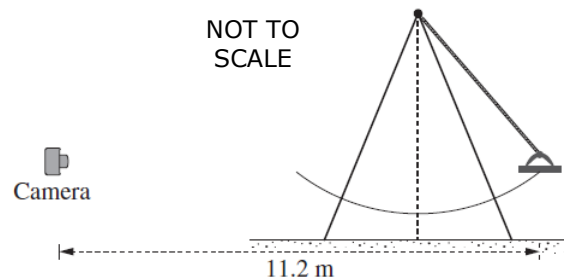
(a) The rate of change of x can be modelled by the equation $\dfrac{dx}{dt} = -1.5\pi \sin\left(\dfrac{5\pi}{4}t\right)$. **2**
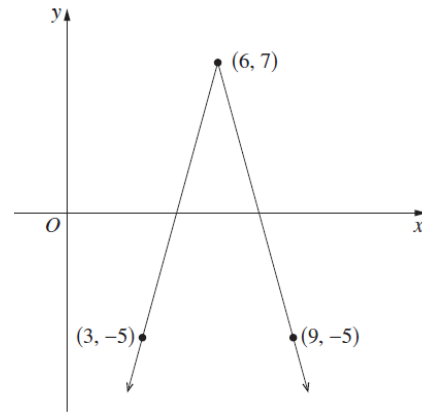
   Find an expression for $x(t)$.

(b) How many times does the swing reach the closest point to the camera during the first 10 seconds? **2**

NESA 2023 Mathematics Advanced HSC Examination

23 MA | **27** | The graph of $y = f(x)$, where $f(x) = a|x - b| + c$, passes through the points $(3, -5)$, $(6, 7)$ and $(9, -5)$ as shown in the diagram.

(a) Find the values of a, b and c. **3**

(b) The line $y = mx$ cuts the graph of $y = f(x)$ in two distinct places.
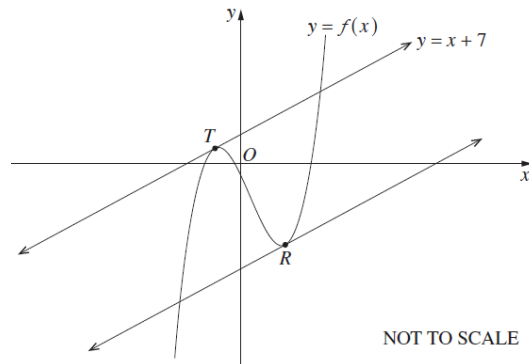Find all possible values of m. **2**

NESA 2023 Mathematics Advanced HSC Examination

23 MA | **28** | The curve $y = f(x)$ is shown on the diagram. The equation of the tangent to the curve at point $T(-1, 6)$ is $y = x + 7$. At a point R, another tangent parallel to the tangent at T is drawn.
The gradient function of the curve is given by
$$\frac{dy}{dx} = 3x^2 - 6x - 8.$$
Find the coordinates of R. **4**

NOT TO SCALE

NESA 2023 Mathematics Advanced HSC Examination

23 MA | **29** | A continuous random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} 12x^2(1 - x), & \text{for } 0 \le x \le 1 \\ 0, & \text{for all other values of } x \end{cases}$$

(a) Find the mode of X. **2**

(b) Find the cumulative distribution function for the given probability density function. **2**

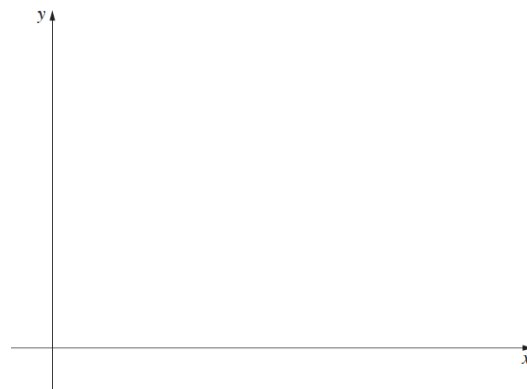(c) Without calculating the median, show that the mode is greater than the median. **2**

NESA 2023 Mathematics Advanced HSC Examination

23 MA | **30** | Let $f(x) = e^{-x \sin x}$.

(a) Find the coordinates of the stationary points of $f(x)$ for $0 \le x \le 2\pi$.
You do NOT need to check the nature of the stationary points. **3**

(b) Without using any further calculus, sketch the graph of $f(x)$ for $0 \le x \le 2\pi$, showing stationary points and intercepts. **2**

NESA 2023 Mathematics Advanced HSC Examination

23 MA **31** Four Year 12 students want to organise a graduation party. All four students have the same probability, $P(F)$, of being available next Friday. All four students have the same probability, $P(S)$, of being available next Saturday.

It is given that $P(F) = \dfrac{3}{10}$, $P(S|F) = \dfrac{1}{3}$, and $P(F|S) = \dfrac{1}{8}$.

Kim is one of the four students.

(a) Is Kim's availability next Friday independent from his availability next Saturday? Justify your answer.   **1**

(b) Show that the probability that Kim is available next Saturday is $\dfrac{4}{5}$.   **2**

(c) What is the probability that at least one of the four students is NOT available next Saturday?   **2**

NESA 2023 Mathematics Advanced HSC Examination

23 MA **32** The curves $y = e^{-2x}$ and $y = e^{-x} - \dfrac{1}{4}$

intersect at exactly one point as shown in the diagram. The point of intersection has

coordinates $\left(\ln 2, \dfrac{1}{4}\right)$.
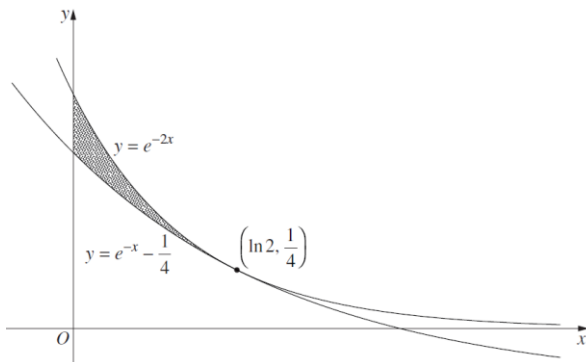

(Do NOT prove this.)

(a) Show that the area bounded by the two curves and the y-axis, as shaded in the diagram, is $\dfrac{1}{4}\ln 2 - \dfrac{1}{8}$.   **3**

(b) Find the values of k such that the curves $y = e^{-2x}$ and $y = e^{-x} + k$ intersect at two points.   **3**

NESA 2023 Mathematics Advanced HSC Examination