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MATHEMATICS ADVANCED

HSC Exam* Questions by Topic 2019 - 2015

v2020

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Year 11 Course

Functions

F1.1 Algebraic techniques

F1.2 Introduction to functions

F1.3 Linear, quadratic & cubic functions

F1.4 Further functions & relations

Trigonometric Functions

T1.1 Trigonometry

T1.2 Radians

T2 Trigonometric functions & identities

Calculus

C1.1 Gradients of tangents

C1.2 Difference quotients

C1.3 The derivative function & its graph

C1.4 Calculating with derivatives

Exponential & Logarithmic Functions

E1.1 Introducing logarithms

E1.2 Logarithmic laws & applications

E1.3 Exponential function & natural logs

E1.4 Graphs & apps of exp & log functions

Statistical Analysis

S1.1 Probability & Venn diagrams

S1.2 Discrete probability distributions

Year 12 Course

Functions

F2 Graphing techniques

Trigonometric Functions

T3 Trigonometric functions and graphs

Calculus

C2.1 Diff of trig, exp & log functions

C2.2 Rules of differentiation

C3.1 The first & second derivatives

C3.2 Applications of the derivative

C4.1 The anti-derivative

C4.2 Areas & the definite integral

Financial Mathematics

M1.1 Modelling investments & loans

M1.2 Arithmetic sequences & series

M1.3 Geometric sequences & series

M1.4 Financial apps of sequences & series

Statistical Analysis

S2.1 Data and summary statistics

S2.2 Bivariate data analysis

S3.1 Continuous random variables

S3.2 The normal distribution

Mathematics Advanced, Ext 1, Ext 2 Reference Sheet (2020 HSC)

Questions by Topic from ...

- **2019 – 2015 Mathematics HSC**
- **NESA Sample examination questions [MA SQ]**
- **NESA Topic Guidance questions [TG]**
- **Selected NESA Maths Stand 2 Sample exam questions [MS SQ] (common topics)**
- **Selected Qs from 2019 – 2015 Maths Extension 1 and 2019 – 2015 Maths Stand 2/General HSCs**

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Year 11: Trigonometric Functions
T1.1 Trigonometry

Back

Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017

Students:

- use the sine, cosine and tangent ratios to solve problems involving right-angled triangles where angles are measured in degrees, or degrees and minutes
- establish and use the sine rule, cosine rule and the area of a triangle formula for solving problems where angles are measured in degrees, or degrees and minutes **AAM**
- find angles and sides involving the ambiguous case of the sine rule
  - use technology and/or geometric construction to investigate the ambiguous case of the sine rule when finding an angle, and the condition for it to arise
- solve problems involving the use of trigonometry in two and three dimensions **AAM**
  - interpret information about a two or three-dimensional context given in diagrammatic or written form and construct diagrams where required
- solve practical problems involving Pythagoras' theorem and the trigonometry of triangles, which may involve the ambiguous case, including finding and using angles of elevation and depression and the use of true bearings and compass bearings in navigation **AAM**

STANDA

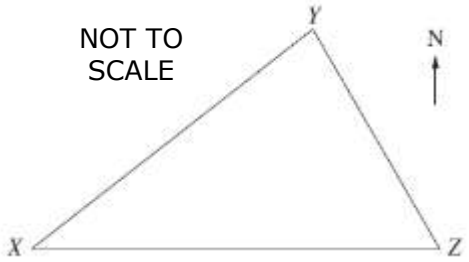
STANDARD

Reference Sheet

MA SQ **12** **Band 2-5**

Solution

The diagram shows the three towns *X*, *Y* and *Z*. Town *Z* is due east of Town *X*. The bearing of Town *Y* from Town *X* is N39°E and the bearing of Town *Z* from Town *Y* is S51°E. The distance between Town *X* and Town *Y* is 1330 km.



A plane flies between the three towns.

(a) Mark the given information on the diagram and explain why $\angle XYZ$ is 90°. **2**

(b) Find the distance between Town *X* and Town *Z* to the nearest kilometre. **2**

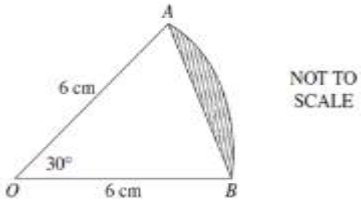
(c) The plane is going to fly from Town *Y* to Town *X*, stopping at Town *Z* on the way. Leaving Town *Y*, the pilot incorrectly sets the bearing of Town *Z* to S50°E. The pilot flies for 1650 km before realising the mistake, then changes course and flies directly to Town *X* without going to Town *Z*.
Which is closer to Town *X*: Town *Z* or the point where the pilot changes course? Justify your answer. **3**

NESA Mathematics Advanced Sample examination materials

TG **1**

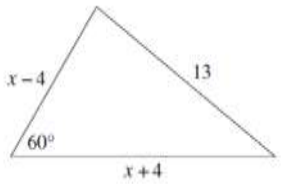
Solution

In the diagram, *OAB* is a sector of the circle with centre *O* and radius 6 cm, where $\angle AOB = 30°$. Determine the exact value of the area of the triangle *OAB.*



NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

TG **2** **17** **M** **13** **a** Find the value of x in the following diagram.



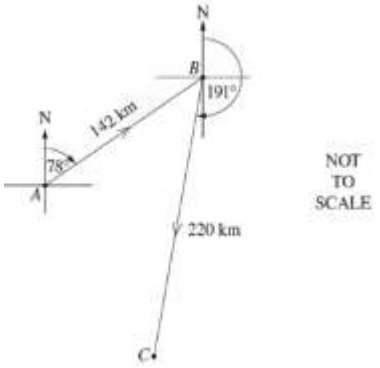
Solution

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions
NESA 2017 Mathematics HSC Examination

TG **3** **14** **M** **13** **d** Chris leaves island A in a boat and sails 142 km on a bearing of 078° to island B. Chris then sails on a bearing of 191° for 220 km to island C, as shown in the diagram.

(a) Show that the distance from island C to island A is approximately 210 km.

(b) Chris wants to sail from island C directly to island A. On what bearing should Chris sail? Give your answer correct to the nearest degree.



Solution

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions
NESA 2014 Mathematics HSC Examination

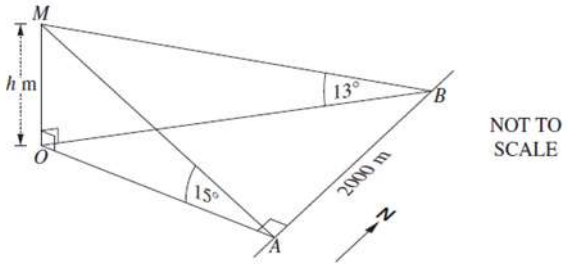
TG **4** Determine the possible dimensions for triangle ABC given AB = 5.4 cm, $\angle BAC$ = 32° and BC = 3 cm.

Solution

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

TG **5** A person walks 2000 metres due north along a road from point A to point B. The point A is due east of a mountain OM, where M is the top of the mountain. The point O is directly below point M and is on the same horizontal plane as the road. The height of the mountain above point O is h metres. From point A, the angle of elevation to the top of the mountain is 15°. From point B, the angle of elevation to the top of the mountain is 13°.

Determine the height of the mountain



Solution

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

TG **6** The Eiffel Tower is located in Paris, a city built on a flat floodplain. Three tourists A, B and C are observing the Eiffel Tower from the ground. A is due north of the tower, C is due east of the tower, and B is on the line-of-sight from A to C and between them. The angles of elevation to the top of the Eiffel Tower from A, B and C are 26°, 28° and 30°, respectively. Determine the bearing of B from the Eiffel Tower.

Solution

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

MS SQ **ME** **4** Which of the following expresses S20°W as a true bearing? **1**

A. 020° B. 070° C. 160° D. 200°

Solution

NESA Mathematics Standard 2 Sample examination materials

MS SQ **ME** **7** Abbey walks 2 km due west from home to a coffee shop. **1**

She then walks on a bearing of 148° to school, which is due south of her home.

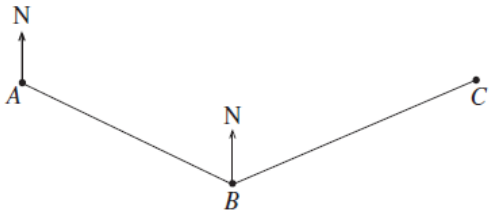
How far south, to the nearest 0.1 km, is Abbey from home?

A. 1.1 km B. 1.2 km C. 3.2 km D. 3.8 km

Solution

NESA Mathematics Standard 2 Sample examination materials

MS SQ ME 8 Paul travels from *A* to *B* on a bearing of 150°.
He then turns and walks to *C* on a bearing of 055°.
What is the size of $\angle ABC$?
A. 85° B. 90°
C. 95° D. 115°

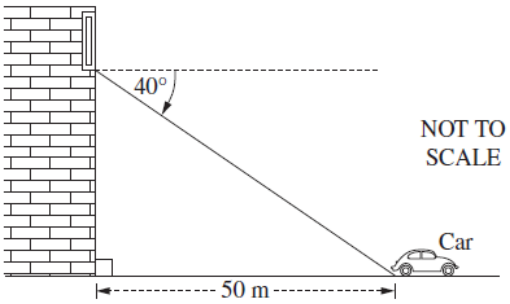


NOT TO SCALE

1 Solution

NESA Mathematics Standard 2 Sample examination materials

MS SQ ME 9 The angle of depression from a window to a car on the ground is 40°.
The car is 50 metres from the base of the building.
How high above the ground is the window, correct to the nearest metre?
A. 32 m
B. 38 m
C. 42 m
D. 48 m

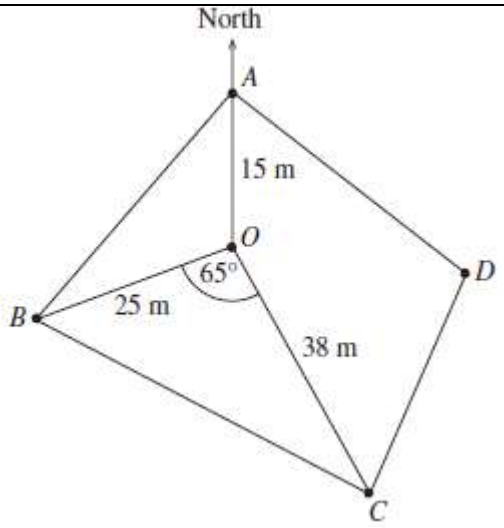


1 Solution

NESA Mathematics Standard 2 Sample examination materials

MS SQ ME 24 The diagram shows the radial survey of a piece of land.
(a) *B* is south west of *O*.
What is the true bearing of *C* from *O*? **2**
(b) What is the area of angle of ΔAOB, to the nearest m^2? **3**

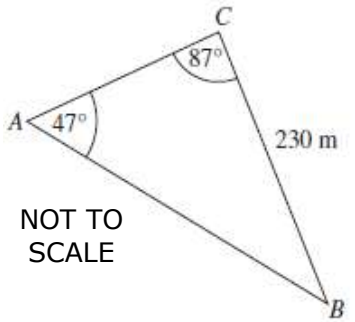
NOT TO SCALE



Solution

NESA Mathematics Standard 2 Sample examination materials

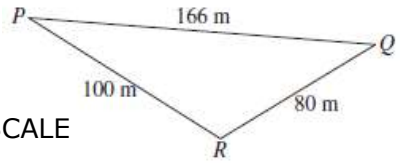
MS SQ ME 25 Lisa owns a piece of land as shown in the diagram.
The length of *BC* is 230 metres.
The size of angle *BCA* is 87° and of angle *BAC* is 47°.
Lisa wants to build a fence along *AC*.
Fencing can be purchased in metre lengths at a cost of $65 per metre.
Calculate the cost of the fencing required.



4 Solution

NESA Mathematics Standard 2 Sample examination materials

MS SQ ME 26 Find the area of triangle PQR, correct to the nearest square metre.



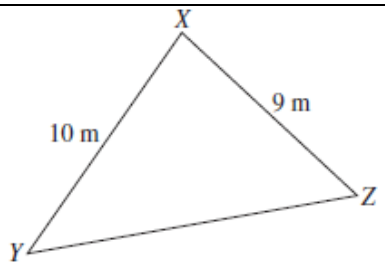
4 Solution

NESA Mathematics Standard 2 Sample examination materials

MS SQ 27 **ME**

The diagram shows triangle XYZ.
The area of the triangle 43 m^2 and $\angle YXZ$ is acute.
What is the size of $\angle YXZ$, to the nearest degree?

NOT TO SCALE



3 Solution

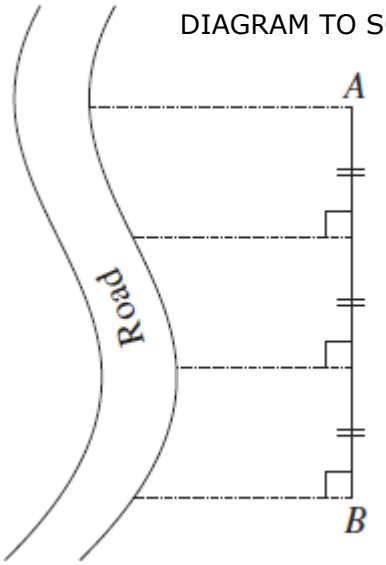
NESA Mathematics Standard 2 Sample examination materials

MS SQ 28 **ME**

The scale diagram shows the aerial view of a block of land bounded on one side by a road. The length of the block, AB, is known to be 45 metres.
Calculate the approximate area of the block of land, using three applications of the trapezoidal rule.

[A note to students from *projectmaths*: Use a ruler to measure AB as 4.5 cm]

DIAGRAM TO SCALE

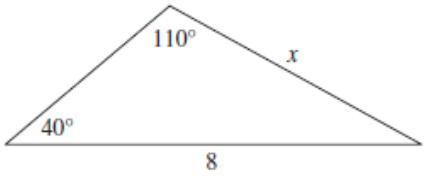


3 Solution

NESA Mathematics Standard 2 Sample examination materials

19 M 11 a

Using the sine rule, find the value of x correct to one decimal place.

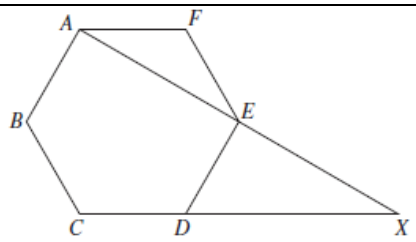


2 Solution

NESA 2019 Mathematics HSC Examination

19 M 14 c

The regular hexagon $ABCDEF$ has sides of length 1.
The diagonal AE and the side CD are produced to meet at the point X.
Copy or trace the diagram into your writing booklet.
Find the exact length of the line segment EX, justifying your answer.



3 Solution

NESA 2019 Mathematics HSC Examination

19 MS 2 4

Which compass bearing is the same as a true bearing of 110°?
A. S20°E B. S20°W C. S70°E D. S70°W

1 Solution

NESA 2019 Mathematics Standard 2 HSC Examination

19 MS 2 12

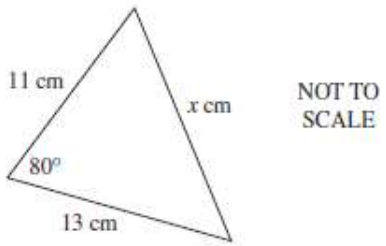
An owl is 7 metres above ground level, in a tree. The owl sees a mouse on the ground at an angle of depression of 32°. How far must the owl fly in a straight line to catch the mouse, assuming the mouse does not move?
A. 3.7 m B. 5.9 m C. 8.3 m D. 13.2 m

1 Solution

NESA 2019 Mathematics Standard 2 HSC Examination

19 MS 2 **17** The diagram shows a triangle with sides of length x cm, 11 cm and 13 cm and an angle of 80°.

Use the cosine rule to calculate the value of x, correct to two significant figures.



3 Solution

NESA 2019 Mathematics Standard 2 HSC Examination

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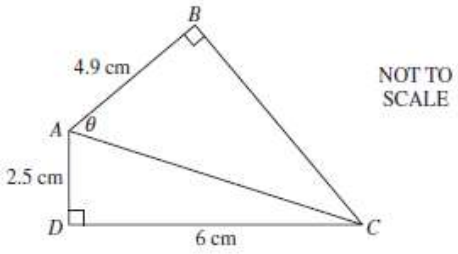
19 MS 2 **22** Two right-angled triangles, ABC and ADC, are shown.

Calculate the size of angle θ, correct to the nearest minute.



3 Solution

NESA 2019 Mathematics Standard 2 HSC Examination

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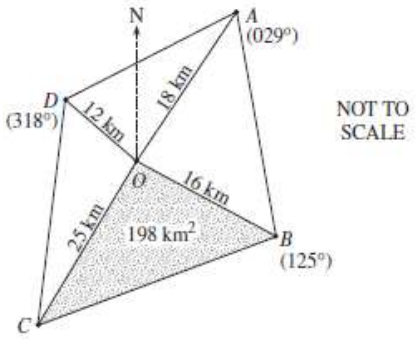
19 MS 2 **35** A compass radial survey shows the positions of four towns A, B, C and D relative to the point O.

The area of the triangle BOC is 198 km^2.

Calculate the bearing of town C from point O, correct to the nearest degree.



3 Solution

NESA 2019 Mathematics Standard 2 HSC Examination

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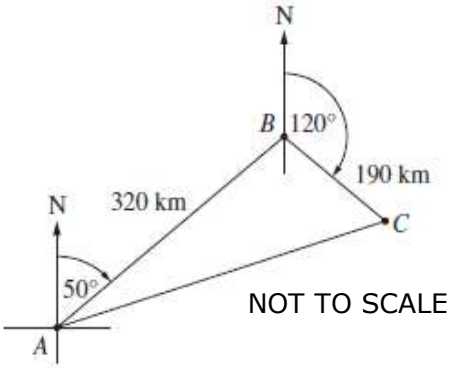
18 M **12 a** A ship travels from Port A on a bearing of 050° for 320 km to Port B. It then travels on a bearing of 120° from 190 km to Port C.

(i) What is the size of $\angle ABC$? **1**

(ii) What is the distance from Part A to Port C? Answer to the nearest kilometre. **2**



Solution

NESA 2018 Mathematics HSC Examination

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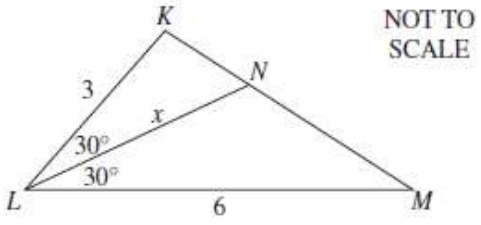
18 M **14 a** In ΔKLM, KL has length 3, LM has length 6 and $\angle KLM$ is 60°. The point N is chosen on side KM so that LN bisects $\angle KLM$.
The length LN is x.

(i) Find the exact value of the area of ΔKLM. **1**

(ii) Hence, or otherwise, find the exact value of x. **2**



Solution

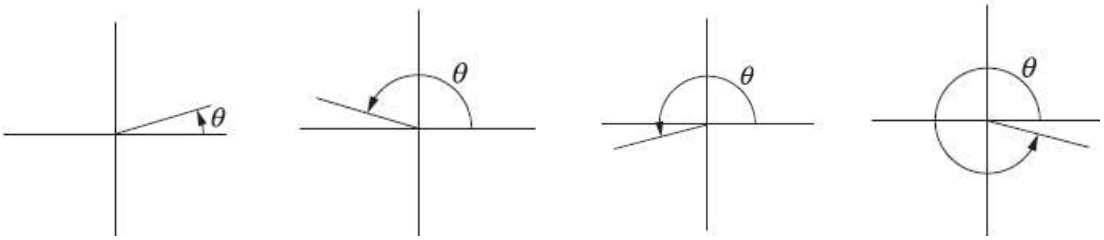
NESA 2018 Mathematics HSC Examination

16 M **1** **1** [Solution]

For the angle θ, $\sin\theta = \frac{7}{25}$ and $\cos\theta = -\frac{24}{25}$.

Which diagram best shows the angle θ?

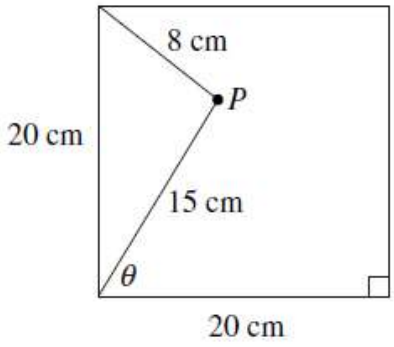
(A) (B) (C) (D)



NESA 2016 Mathematics HSC Examination

16 M **12 c** **3** [Solution]

Square tiles of side length 20 cm are being used to tile a bathroom.
The tiler needs to drill a hole in one of the tiles at a point P which is 8 cm from one corner and 15 cm from an adjacent corner.
To locate the point *P* the tiler needs to know the size of the angle θ shown in the diagram.
Find the size of the angle θ to the nearest degree.



NOT TO SCALE

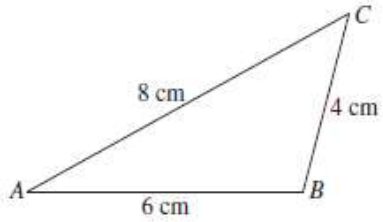
NESA 2016 Mathematics HSC Examination

15 M **13 a** [Solution]

The diagram shows ΔABC with sides $AB = 6$ cm, $BC = 4$ cm and $AC = 8$ cm.

(i) Show that $\cos A = \frac{7}{8}$. **1**

(ii) By finding the exact value of $\sin A$, determine the exact value of the area of ΔABC. **2**



Not to scale

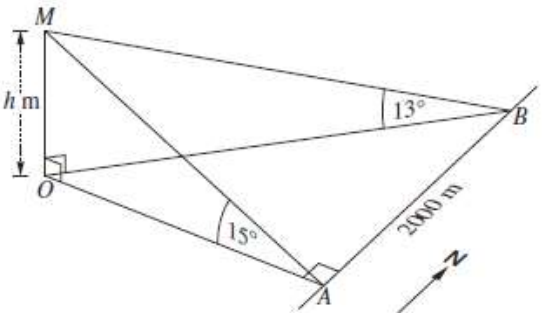
NESA 2016 Mathematics HSC Examination

15 MX **12 c** [Solution]

A person walks 2000 metres due north along a road from point *A* to point *B*. The point *A* is due east of a mountain *OM*, where *M* is the top of the mountain. The point *O* is directly below point *M* and is on the same horizontal plane as the road. The height of the mountain above point *O* is *h* metres. From point *A*, the angle of elevation to the top of the mountain is 15°. From point *B*, the angle of elevation to the top of the mountain is 13°.

(i) Show that $OA = h\cot 15°$. **1**

(ii) Hence, find the value of *h*. **2**



Not to scale

NESA 2015 Mathematics Extension 1 HSC Examination

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Year 11: Statistical Analysis
S1.1 Probability and Venn Diagrams

Back

Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017

Students:

- understand and use the concepts and language associated with theoretical probability, relative frequency and the probability scale
- solve problems involving simulations or trials of experiments in a variety of contexts **AAM**
  - identify factors that could complicate the simulation of real-world events (ACMEM153)
  - use relative frequencies obtained from data as point estimates of probabilities (ACMMM055)
- use arrays and tree diagrams to determine the outcomes and probabilities for multi-stage experiments (ACMEM156) **AAM**
- use Venn diagrams, set language and notation for events, including $\bar{A}$ (or A' or A^c) for the complement of an event A, $A \cap B$ for 'A and B', the intersection of events A and B, and $A \cup B$ for 'A or B', the union of events A and B, and recognise mutually exclusive events (ACMMM050) **AAM**
  - use everyday occurrences to illustrate set descriptions and representations of events and set operations (ACMMM051)
- establish and use the rules: $P(\bar{A}) = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (ACMMM054) **AAM**
- understand the notion of conditional probability and recognise and use language that indicates conditionality (ACMMM056)
- use the notation $P(A|B)$ and the formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$, $P(B) \neq 0$ for conditional probability (ACMMM057) **AAM**
- understand the notion of independence of an event A from an event B, as defined by $P(A|B) = P(A)$ (ACMMM058)
- use the multiplication law $P(A \cap B) = P(A)P(B)$ for independent events A and B and recognise the symmetry of independence in simple probability situations (ACMMM059)

STANDARD 2

ST

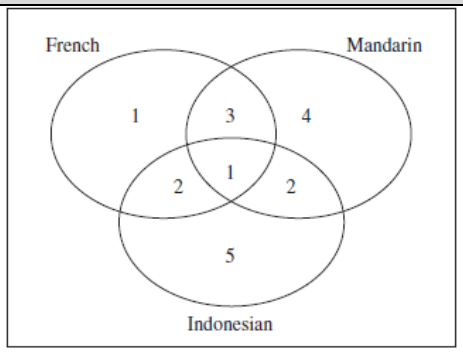
Reference Sheet

MA SQ **5** **Band 3-4**

In a mixed language class, students study French, Mandarin and Indonesian. The number of students who study each language are shown in the Venn diagram.

A student who studies Indonesian is selected at random.

What is the probability that he/she also studies French?



A. $\frac{1}{6}$ B. $\frac{3}{10}$

C. $\frac{3}{5}$ D. $\frac{7}{10}$

1 Solution

NESA Mathematics Advanced Sample examination materials

MA SQ **16** **Band 2-5** A survey found that in a large population approximately 20% of people are left-handed. **Solution**

(a) Three people are selected at random. **2**
Find the probability that at least one of them is left-handed.

(b) What is the smallest number of people that would need to be selected to have a greater than 99% chance that at least one of them is left-handed? **3**

NESA Mathematics Advanced Sample examination materials

TG **1** Six girls' names and five boys' names are placed in a hat. **Solution**
Two names are drawn without replacement.
What is the probability that a girl's and a boy's name are chosen?

NESA Mathematics Advanced Year 11 Topic Guide: Statistical analysis

TG **2** In a raffle, 30 tickets are sold and there are two prizes. **Solution**
John buys five tickets.
What is the probability that John wins at least one prize?

NESA Mathematics Advanced Year 11 Topic Guide: Statistical analysis

TG **3** In Australia, approximately 9% of the population has the blood type O negative. **Solution**
If three people are chosen at random from the population, find the probability that:
(a) none has O negative blood
(b) at least one has O negative blood.

NESA Mathematics Advanced Year 11 Topic Guide: Statistical analysis

TG **4** **Solution**

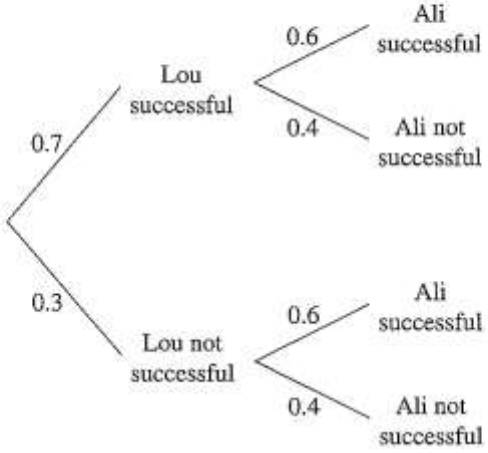
The manager of a team notices that the team has a probability of $\frac{2}{3}$ of winning the game if it is raining and if it is dry, the probability of the team winning is $\frac{1}{5}$. The probability that it will rain on a day when they play is $\frac{1}{4}$.

(a) Find the probability that they will not win.
(b) Given that the team has won a game, calculate the probability that it rained on the day of the match.

NESA Mathematics Advanced Year 11 Topic Guide: Statistical analysis

TG **5** **Solution**

10 MG **20** Lou and Ali are on a fitness program for one month.
The probability that Lou will finish the program successfully is 0.7 while the probability that Ali will finish successfully is 0.6.
The probability tree diagram shows this information.



What is the probability that only one of Lou and Ali will be successful?

NESA Mathematics Advanced Year 11 Topic Guide: Statistical analysis

TG **6** A bag contains two red balls, one black ball, and one white ball. **Solution**
Andrew selects one ball from the bag and keeps it hidden.
He then selects a second ball, also keeping it hidden.
(a) Draw a tree diagram to show all the possible outcomes.
(b) Find the probability that both the selected balls are red.
(c) Find the probability that at least one of the selected balls is red.
(d) Andrew drops one of the selected balls and we can see that it is red.
What is the probability that the ball that is still hidden is also red?

NESA Mathematics Advanced Year 11 Topic Guide: Statistical analysis

19 M **6** A game is played by tossing an ordinary 6-sided die and an ordinary coin at the same time. The game is won if the uppermost face of the die shows an even number or the uppermost face of the coin shows a tail (or both). What is the probability of winning this game? **1** Solution

A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. $\frac{3}{4}$ D. 1

NESA 2019 Mathematics HSC Examination

19 M **11 f** A bag contains 5 green beads and 7 purple beads. Two beads are selected at random, without replacement. What is the probability that the two beads are the same colour? **2** Solution

NESA 2019 Mathematics HSC Examination

19 M **15 d** The probability that a person chosen at random has red hair is 0.02. Solution

(i) Two people are chosen at random. **2**

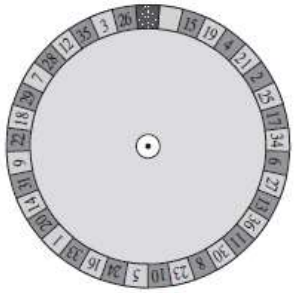
What is the probability that at least ONE has red hair?

(ii) What is the smallest number of people that can be chosen at random so that the probability that at least ONE has red hair is greater than 0.4 **2**

NESA 2019 Mathematics HSC Examination

19 MS 2 **20** A roulette wheel has the numbers 0, 1, 2, …, 36 where each of the 37 numbers is equally likely to be spun. **2** Solution

If the wheel is spun 18 500 times, calculate the expected frequency of spinning the number 8.



NESA 2019 Mathematics Standard 2 HSC Examination

19 MS 2 **25** A bowl of fruit contains 17 apples of which 9 are red and 8 are green. Dennis takes one apple at random and eats it. Margaret also takes an apple at random and eats it. By drawing a probability tree diagram, or otherwise, find the probability that Dennis and Margaret eat apples of the same colour. **3** Solution

NESA 2019 Mathematics Standard 2 HSC Examination

18 M **6** A runner has four different pairs of shoes. If two shoes are selected at random, what is the probability that they will be a matching pair? **1** Solution

A. $\frac{1}{56}$ B. $\frac{1}{16}$ C. $\frac{1}{7}$ D. $\frac{1}{4}$

NESA 2018 Mathematics HSC Examination

18 M **14 e** Two machines, *A* and *B*, produce pens. It is known that 10% of the pens produced by machine *A* are faulty and that 5% of the pens produced by machine *B* are faulty. Solution

(i) One pen is chosen at random from each machine. What is the probability that at least one of the pens is faulty? **1**

(ii) A coin is tossed to select one of the two machines. Two pens are chosen at random from the selected machine. What is the probability that neither pen is faulty? **2**

NESA 2018 Mathematics HSC Examination

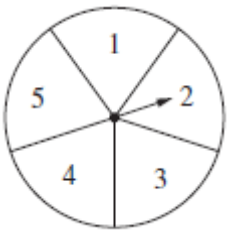
18 M **16 b** A game involves rolling two six-sided dice, followed by rolling a third six-sided die. To win the game, the number rolled on the third die must lie between the two numbers rolled previously. For example, if the first two dice show 1 and 4, the game can only be won by rolling a 2 or 3 with the third die. **Solution**

(i) What is the probability that a player has no chance of winning before rolling the third die? **2**

(ii) What is the probability that a player wins the game? **2**

NESA 2018 Mathematics HSC Examination

17 M **12 e** A spinner is marked with the numbers 1, 2, 3, 4 and 5. When it is spun, each of the five numbers is equally likely to occur. The spinner is spun three times. **Solution**



(i) What is the probability that an even number occurs on the first spin? **1**

(ii) What is the probability that an even number occurs on at least one of the three spins? **1**

(iii) What is the probability that an even number occurs on the first spin and odd numbers occur on the second and third spins? **1**

(iv) What is the probability that an even number occurs on exactly one of the three spins? **1**

NESA 2017 Mathematics HSC Examination

16 M **2** In a raffle, 30 tickets are sold and there is one prize to be won. What is the probability that someone buying 6 tickets wins the prize? **1** **Solution**

(A) $\frac{1}{30}$ (B) $\frac{1}{6}$ (C) $\frac{1}{5}$ (D) $\frac{1}{4}$

NESA 2016 Mathematics HSC Examination

16 M **15 b** An eight-sided die is marked with numbers 1, 2, …, 8. A game is played by rolling the die until an 8 appears on the uppermost face. At this point the game ends. **Solution**

(i) Using a tree diagram, or otherwise, explain why the probability of the game ending before the fourth roll is $\frac{1}{8} + \frac{7}{8} \times \frac{1}{8} + \left(\frac{7}{8}\right)^2 \times \frac{1}{8}$. **2**

(ii) What is the smallest value of n for which the probability of the game ending before the nth roll is more than $\frac{3}{4}$? **3**

NESA 2016 Mathematics HSC Examination

15 M **4** The probability that Mel's soccer team wins this weekend is $\frac{5}{7}$. The probability that Mel's rugby league team wins this weekend is $\frac{2}{3}$. What is the probability that neither team wins this weekend? **1** **Solution**

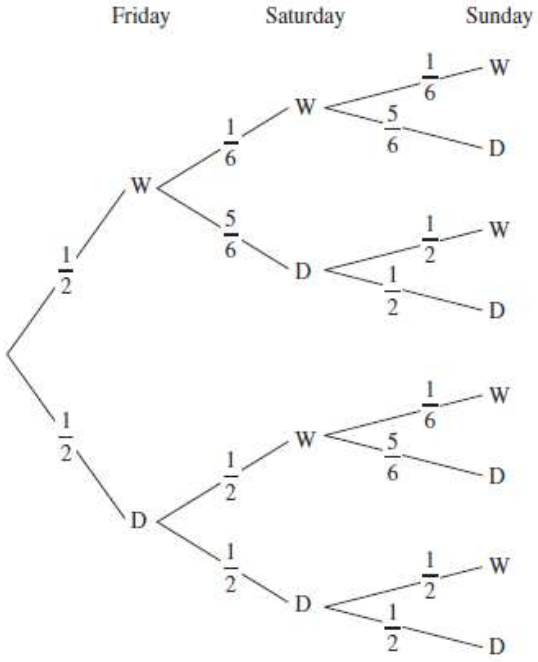
(A) $\frac{2}{21}$ (B) $\frac{10}{21}$ (C) $\frac{13}{21}$ (D) $\frac{19}{21}$

NESA 2015 Mathematics HSC Examination

15 M **14 b** Weather records for a town suggest that: **2** Solution

- if a particular day is wet (*W*), the probability of the next day being dry is $\frac{5}{6}$. **2**
- if a particular day is dry (*D*), the probability of the next day being dry is $\frac{1}{2}$.

In a specific week Thursday is dry. The tree diagram shows the possible outcomes for the next three days: Friday, Saturday and Sunday.



(i) Show that the probability of Saturday being dry is $\frac{2}{3}$. **1**

(ii) What is the probability of both Saturday and Sunday being wet? **2**

(iii) What is the probability of at least one of Saturday and Sunday being dry? **1**

NESA 2015 Mathematics HSC Examination

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Year 12: Calculus
C3.1 The First and Second Derivatives

Back

Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017

Students:

- use the first derivative to investigate the shape of the graph of a function
  - deduce from the sign of the first derivative whether a function is increasing, decreasing or stationary at a given point or in a given interval
  - use the first derivative to find intervals over which a function is increasing or decreasing, and where its stationary points are located
  - use the first derivative to investigate a stationary point of a function over a given domain, classifying it as a local maximum, local minimum or neither
  - determine the greatest or least value of a function over a given domain (if the domain is not given, the natural domain of the function is assumed) and distinguish between local and global minima and maxima
- define and interpret the concept of the second derivative as the rate of change of the first derivative function in a variety of contexts, for example recognise acceleration as the second derivative of displacement with respect to time (ACMMM108, ACMMM109) **AAM**
  - understand the concepts of concavity and points of inflection and their relationship with the second derivative (ACMMM110)
  - use the second derivative to determine concavity and the nature of stationary points
  - understand that when the second derivative is equal to 0 this does not necessarily represent a point of inflection

Reference Sheet

TG **1** Sketch the graph of the function $f(x) = x^3 + 3x^2 - 9x - 9$ by identifying stationary points and determining their nature. **Solution**

NESA Mathematics Advanced Year 12 Topic Guide: Calculus

TG **2** By considering the sign of the first derivative, show that the function $f(x) = \dfrac{1}{3x-2}$ is decreasing throughout its domain. **Solution**

NESA Mathematics Advanced Year 12 Topic Guide: Calculus

TG **3** Consider the curve $y = \frac{1}{4}x^4 - x^2$. **Solution**

(a) Find any stationary points and determine their nature.
(b) Find any points of inflection.
(c) Sketch the curve, indicating where the curve crosses the x-axis.
(d) For what values of x is the curve concave down?

NESA Mathematics Advanced Year 12 Topic Guide: Calculus

19 M **14 b** The derivative of a function $y = f(x)$ is given by $f'(x) = 3x^2 + 2x - 1$. **Solution**

(i) Find the x-values of the two stationary points of $y = f(x)$, and determine the nature of the stationary points. **2**

(ii) The curve passes through the point (0, 4). Find an expression for $f(x)$. **2**

(iii) Hence sketch the curve, clearly indicating the stationary points. **2**

(iv) For what values of x is the curve concave down? **1**

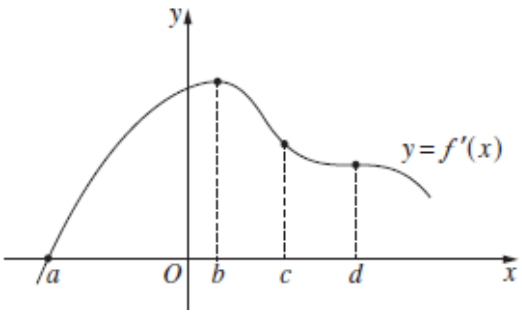
NESA 2019 Mathematics HSC Examination

18 M **9** The diagram shows the graph of $y = f'(x)$, the derivative of a function. **1** Solution

For what value of x does the graph of the function $f(x)$ have a point of inflexion?

A. $x = a$ B. $x = b$

C. $x = c$ D. $x = d$



NESA 2018 Mathematics HSC Examination

18 M **13 a** Consider the curve $y = 6x^2 - x^3$. Solution

(i) Find the stationary points and determine their nature. **3**

(ii) Given that the point (2, 16) lies on the curve, show that it is a point of inflexion. **2**

(iii) Sketch the curve, showing the stationary points, the point of inflexion and the x and y intercepts. **2**

NESA 2018 Mathematics HSC Examination

18 M **14 c** Let $f(x) = x^3 + kx^2 + 3x - 5$, where k is a constant. **3** Solution

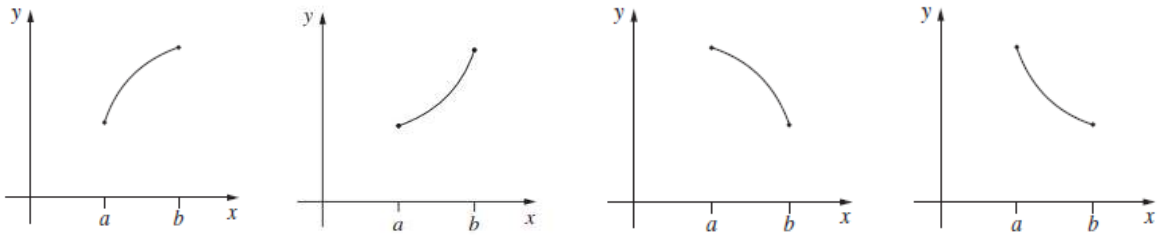
Find the values of k for which $f(x)$ has NO stationary points.

NESA 2018 Mathematics HSC Examination

17 M **4** The function $f(x)$ is defined for $a \le x \le b$. On this interval, $f'(x) > 0$ and $f''(x) < 0$. **1** Solution

Which graph best represents $y = f(x)$?

(A) (B) (C) (D)



NESA 2017 Mathematics HSC Examination

17 M **13 b** Consider the curve $y = 2x^3 + 3x^2 - 12x + 7$. Solution

(i) Find the stationary points and determine their nature. **4**

(ii) Sketch the curve, labelling the stationary points. **2**

(iii) Hence, or otherwise, find the values of x for which $\frac{dy}{dx}$ is positive. **1**

NESA 2017 Mathematics HSC Examination

16 M **13 a** Consider the function $y = 4x^3 - x^4$. Solution

(i) Find the two stationary points and determine their nature. **4**

(ii) Sketch the graph of the function, clearly showing the stationary points and the x and y intercepts. **2**

NESA 2016 Mathematics HSC Examination

16 MX 1 **9** **1** Solution

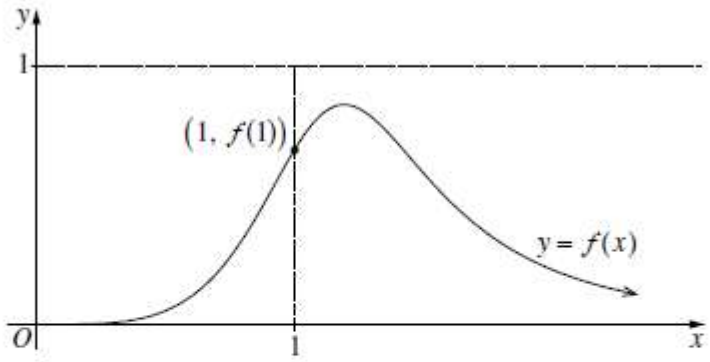
The diagram shows the graph of $y = f(x)$. Which of the following x is a correct statement?

(A) $f''(1) < f(1) < 1 < f'(1)$

(B) $f''(1) < f'(1) < f(1) < 1$

(C) $f(1) < 1 < f'(1) < f''(1)$

(D) $f'(1) < f(1) < 1 < f''(1)$



Not to scale

NESA 2016 Mathematics Extension 1 HSC Examination

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15 M **13** Solution

Consider the curve $y = x^3 - x^2 - x + 3$.

c (i) Find the stationary points and determine their nature. **4**

(ii) Given that the point $P(\frac{1}{3}, \frac{70}{27})$ lies on the curve, prove that there is a point of inflexion at P. **2**

(iii) Sketch the curve, labelling the stationary points, point of inflexion and y-intercept. **2**

NESA 2015 Mathematics HSC Examination

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Year 12: Financial Mathematics
M1.4 Financial Applications of Sequences and Series

Back

Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017

Students:

- use geometric sequences to model and analyse practical problems involving exponential growth and decay (ACMMM076) **AAM**
  - calculate the effective annual rate of interest and use results to compare investment returns and cost of loans when interest is paid or charged daily, monthly, quarterly or six-monthly (ACMGM095)
  - solve problems involving compound interest loans or investments, eg determining the future value of an investment or loan, the number of compounding periods for an investment to exceed a given value and/or the interest rate needed for an investment to exceed a given value (ACMGM096)
  - recognise a reducing balance loan as a compound interest loan with periodic repayments, and solve problems including the amount owing on a reducing balance loan after each payment is made
- solve problems involving financial decisions, including a home loan, a savings account, a car loan or superannuation **AAM**
  - calculate the future value or present value of an annuity by developing an expression for the sum of the calculated compounded values of each contribution and using the formula for the sum of the first n terms of a geometric sequence
  - verify entries in tables of future values or annuities by using geometric series

Reference Sheet

STANDARD 2

MA SQ **2** Band 3-6 — What amount must be invested now at 4% per annum, compounded quarterly, so that in five years it will have grown to $60 000? **1** Solution

A. $8919 B. $11 156 C. $49 173 D. $49 316

NESA Mathematics Advanced Sample examination materials

MS SQ **FM 11** — Mia wants to invest $42 000 for a total of 5 years. She has three investment options. **5** Solution

Option A – simple interest is paid at the rate of 6% per annum
Option B – compound interest is paid at a rate of 5.5% per annum, compounded annually
Option C – compound interest is paid at a rate of 4.8% per annum, compounded quarterly
Determine Mia's best investment option. Support your answer with calculations.

NESA Mathematics Standard 2 Sample examination materials

MS SQ **FM 19** — A house was purchased at the start of 1986 for $45 000. **2** Solution

Assume that the value of the house has increased by 8% per annum since then.
What is the value of the house at the end of 2019, to the nearest $1000?

NESA Mathematics Standard 2 Sample examination materials

MS SQ 7 **FM**

A sum of $250 000 was borrowed to buy a house. The interest rate and monthly repayment for the loan are shown in the spreadsheet.

What is the total interest charged for the first four months of this loan?

A. $6364.32
B. $6366.11
C. $6369.67
D. $6376.25

	A	B	C	D	E
1	**Home Loan Table**			*This table assumes the same number of days in each month, ie*	
2		Amount = $250 000			
3		Annual Interest Rate = 7.65%		Interest = Rate/12 × Principal	
4		Monthly Repayment (*R*) = $1871.94			
5					
6	**Month**	**Principal (*P*)**	**Interest (*I*)**	***P* + *I***	***P* + *I* − *R***
7	1	$250 000.00	$1593.75	$251 593.75	$249 721.81
8	2	$249 721.81	$1591.98	$251 313.79	$249 441.85
9	3	$249 441.85	$1590.19	$251 032.04	
10	4				

Sheet 1 Sheet 2

NESA Mathematics Standard 2 Sample examination materials

1 Solution

MS SQ 12 **FM**

A person takes out a loan of $29 500 at 8% per annum interest for four years.

Monthly repayments for loans at different interest rates are shown in the spreadsheet.

How much interest does the person pay over the term of this loan?

	A	B	C	D	E
1		**Monthly repayments**			
2		**Term of loan:** 4 years			
3					
4	**Amount**	**Interest rate p.a.**			
5	**borrowed**	6%	7%	8%	9%
6	$27 000	$634.10	$646.55	$659.15	$671.90
7	$27 500	$645.84	$658.52	$671.36	$684.34
8	$28 000	$657.58	$670.49	$683.56	$696.78
9	$28 500	$669.32	$682.47	$695.77	$709.22
10	$29 000	$681.07	$694.44	$707.97	$721.67
11	$29 500	$692.81	$706.41	$720.18	$734.11
12	$30 000	$704.55	$718.39	$732.39	$746.55

Sheet 1 Sheet 2

NESA Mathematics Standard 2 Sample examination materials

2 Solution

MS SQ 13 **FM**

Jay bought a computer for $3600. His friend Julie says that all computers are worth nothing (ie the value is $0) after 3 years.

(a) Assume that Julie is correct. Find the amount that the computer would be worth after 2 years, if the straight-line method of depreciation is used. **2**

(b) Explain why the computer would never be worth nothing if the declining-balance method of depreciation is used, with a 30% per annum rate of depreciation. Use suitable calculations to support your answer. **2**

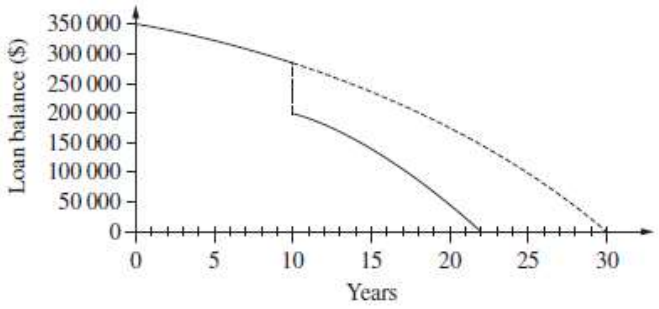
NESA Mathematics Standard 2 Sample examination materials

Solution

MS SQ 14 **FM**

15 MG **29 b**

An electrician borrowed $350 000 to be repaid over 30 years, with monthly repayments of $1880. However, after 10 years he made a lump sum payment of $80 000. The monthly repayment remained unchanged. The graph shows the balances owing over the period of the loan.



Using information from the graph, calculate how much less the electrician paid over the period of the loan by making the lump sum payment.

NESA Mathematics Standard 2 Sample examination materials
NESA 2015 Mathematics General HSC Examination

2 Solution

19 M **16 a** A person wins \$1 000 000 in a competition and decides to invest this money in an account that earns interest at 6% per annum compounded quarterly. The person decides to withdraw \$80 000 from this account at the end of every fourth quarter. **Solution**

Let A be the amount remaining in the account after the nth withdrawal.

(i) Show that the amount remaining in the account after the withdrawal at the end of the eighth quarter is $A_2 = 1\ 000\ 000 \times 1.015^8 - 80\ 000(1 + 1.015^4)$. **2**

(ii) For how many years can the full amount of \$80 000 be withdrawn? **3**

NESA 2019 Mathematics HSC Examination

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18 M **16c** Kara deposits an amount of \$300 000 into an account which pays compound interest of 4% per annum, added to the account at the end of each year. Immediately after the interest is added, Kara makes a withdrawal for expenses for the coming year. The first withdrawal is \$$P$. Each subsequent withdrawal is 5% greater than the previous one. Let \$$A_n$ be the amount in the account after the nth withdrawal. **Solution**

(i) Show that $A_2 = 300\ 000(1.04)^2 - P[(1.04) + (1.05)]$ **1**

(ii) Show that $A_3 = 300\ 000(1.04)^3 - P[(1.04)^2 + (1.04)(1.05) + (1.05)^2]$. **1**

(i) Show that there will be money in the account when $\left(\frac{105}{104}\right)^n < 1 + \frac{3000}{P}$. **3**

NESA 2018 Mathematics HSC Examination

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17 M **15b** Anita opens a savings account. At the start of each month she deposits \$$X$ into the savings account. At the end of the month, after interest is added into the savings account, the bank withdraws \$2500 from the savings account as a loan repayment. Let M_n be the amount in the savings account after the n^{th} withdrawal. The savings account pays interest at 4.2% per annum compounded monthly. **Solution**

(i) Show that after the second withdrawal the amount in the savings account is given by $M_2 = X(1.0035^2 + 1.0035) - 2500(1.0035 + 1)$. **2**

(ii) Find the value of X so that the amount in the savings account is \$80 000 after the last withdrawal of the fourth year. **3**

NESA 2017 Mathematics HSC Examination

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15 M **14c** Sam borrows \$100 000 to be repaid at a reducible interest rate of 0.6% per month. Let \$$A_n$ be the amount owing at the end of n months and \$$M$ be the monthly repayment. **Solution**

(i) Show that $A_2 = 100\ 000(1.006)^2 - M(1 + 1.006)$. **1**

(ii) Show that $A_n = 100\ 000(1.006)^n - M\left(\frac{(1.006)^n - 1}{0.006}\right)$. **2**

(iii) Sam makes monthly repayments of \$780. Show that after making 120 monthly repayments the amount owing is \$68 500 to the nearest \$100. **1**

(iv) Immediately after making the 120th repayment, Sam makes a one-off payment, reducing the amount owing to \$48 500. The interest rate and monthly repayment remain unchanged. After how many more months will the amount owing be completely repaid? **3**

NESA 2015 Mathematics HSC Examination

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $$\alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1$$

$$S = \frac{a}{1 - r}, \quad |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

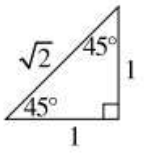
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

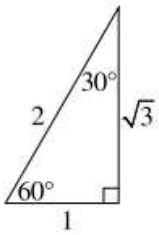
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

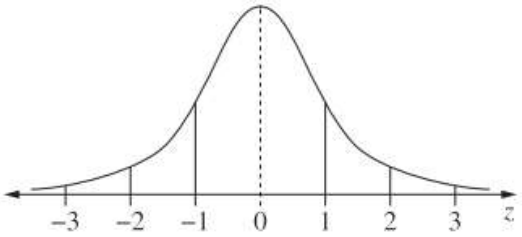
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c \quad \text{where } n \neq -1$$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left\{f(a)+f(b)+2\left[f(x_1)+\cdots+f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^ne^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^2(x-c)$$