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MATHEMATICS EXTENSION 1

HSC Exam* Questions by Topic

2020 - 2016

v2020

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2020 HSC

2020 NESA Sample Paper

Year 11 Course

Functions

F1.1 Graphical relationships

F1.2 Inequalities

F1.3 Inverse functions

F1.4 Parametric form of function or rel.

F2.1 Remainder and factor theorems

F2.2 Sums & products of roots of polyns

Trigonometric Functions

T1 Inverse trigonometric functions

T2 Further trigonometric identities

Calculus

C1.1 Rates of change with respect to time

C1.2 Exponential growth & decay

C1.3 Related rates of change

Combinatorics

A1.1 Permutations and combinations

A1.2 Binomial expansion & Pascal's Δ

Year 12 Course

Proof

P1 Proof by mathematical induction

Vectors

V1.1 Introduction to vectors

V1.2 Further operations with vectors

V1.3 Projectile motion

Trigonometric Functions

T3 Trigonometric equations

Calculus

C2 Further calculus skills

C3.1 Further area and volume of solids

C3.2 Differential equations

Statistical Analysis

S1.1 Bernoulli & binomial distributions

S1.2 Normal approx for the sample prop^{n}

Mathematics Advanced, Ext 1, Ext 2 Reference Sheet (2020 HSC)

Questions by Topic from …

- **2020 – 2016 HSCs (MX1: Mathematics Extension 1, M: Mathematics)**
- **NESA Sample HSC examination [SP]**
- **Additional NESA sample questions [SQ]**
- **NESA Topic Guidance [TG] – selected questions**

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Year 11: Calculus
C1.3 Related rates of change

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https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017

Students:

- solve problems involving related rates of change as instances of the chain rule (ACMSM129) **AAM**
- develop models of contexts where a rate of change of a function can be expressed as a rate of change of a composition of two functions, and to which the chain rule can be applied

Reference Sheet

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20 MX 1 **10** The quantities P, Q and R are connected by the related rates, **1** **Solution**

$$\frac{dR}{dt} = -k^2$$

$$\frac{dP}{dt} = -l^2 \times \frac{dR}{dt}$$

$$\frac{dP}{dt} = m^2 \times \frac{dQ}{dt}$$

where k, l and m are non-zero constants.

Which of the following statements is true?

A P is increasing and Q is increasing B P is increasing and Q is decreasing

C P is decreasing and Q is increasing D P is decreasing and Q is decreasing

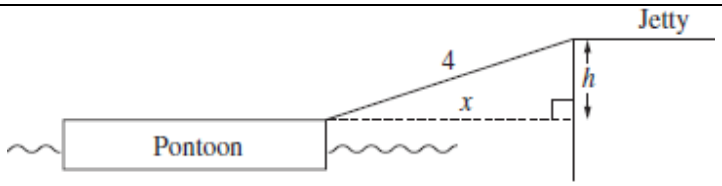
NESA 2020 Mathematics Extension 1 HSC Examination

SP MX 1 **9** A stone drops into a pond, creating a circular ripple. **1** **Solution**
The radius of the ripple increases from 0 cm at a constant rate of 5 cm s^{-1}.
At what rate is the area enclosed within the ripple increasing when the radius is 15 cm?

A. 25π cm^2 s^{-1} B. 30π cm^2 s^{-1} C. 150π cm^2 s^{-1} D. 225π cm^2 s^{-1}

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

TG 04 MX 1 **1** **3c** A ferry wharf consists of a floating pontoon linked to a jetty by a 4 metre long walkway. Let h metres be the difference in height between the top of the pontoon and the top of the jetty and let x metres be the horizontal distance between the pontoon and the jetty. **Solution**



(i) Find an expression for x in terms of h. **1**

(ii) When the top of the pontoon is 1 metre lower than the top of the jetty, the tide is rising at a rate of 0.3 metres per hour. **3**
At what rate is the pontoon moving away from the jetty?

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus
NESA 2004 Mathematics Extension 1 HSC Examination

TG **2** A spherical balloon is being deflated so that the radius decreases at a constant rate of 10 mm per second. Calculate the rate of change of volume when the radius of the balloon is 100 mm. **Solution**

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus

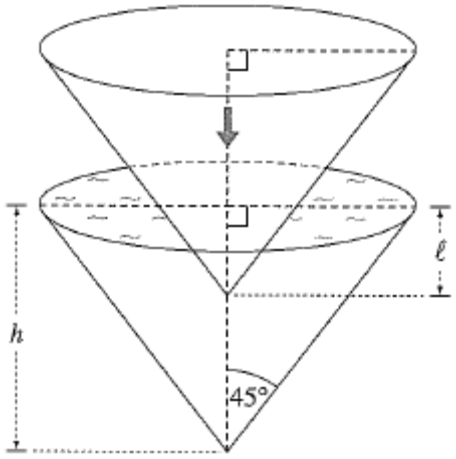
TG **3** A spherical bubble is expanding so that its volume increases at the constant rate of 70 mm^3 per second. What is the rate of increase of its surface area when the radius is 10 mm? **Solution**

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus

TG **4** A hot air balloon is at a constant height of 160 metres above the ground, and moving parallel to the ground, at a speed of 20 metres per minute. Find the rate at which the balloon is moving away from an observer on the ground at the time when the distance from the observer to the balloon is 400 metres. **Solution**

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus

TG **11 MX 1** **5** **7a** The diagram shows two identical circular cones with a common vertical axis. Each cone has height h cm and semi-vertical angle 45°. The lower cone is completely filled with water. The upper cone is lowered vertically into the water as shown in the diagram. The rate at which it its lowered is given by $\frac{d\ell}{dt} = 10$, where ℓ cm is the distance the upper cone has descended into the water after t seconds. As the upper cone is lowered, water spills from the lower cone. The volume of water remaining in the lower cone at time t is V cm^3. **Solution**



Not to scale

(i) Show that $V = \frac{\pi}{3}\left(h^3 - \ell^3\right)$. **1**

(ii) Find the rate at which V is changing with respect to time when $\ell = 2$. **2**

(iii) Find the rate at which V is changing with respect to time when the lower cone has lost $\frac{1}{8}$ of its water. Give your answer in terms of h. **2**

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus
NESA 2011 Mathematics Extension 1 HSC Examination

19 MX 1 **12 a** Distance A is inversely proportional to distance B, such that $A = \frac{9}{B}$, where A and B are measured in metres. The two distances vary with respect to time. Distance B is increasing at a rate of 0.2 ms^{-1}. What is the value of $\frac{dA}{dt}$ when $A = 12$? **3** **Solution**

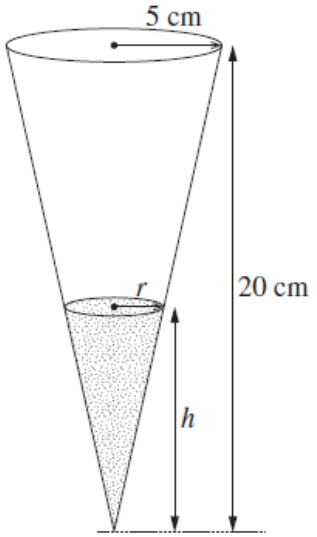
NESA 2019 Mathematics Extension 1 HSC Examination

17 MX 1 **8** A stone drops into a pond, creating a circular ripple. The radius of the ripple increases from 0 cm, at a constant rate of 5 $cm\,s^{-1}$. At what rate is the area of enclosed within the ripple increasing when the radius is 15 cm? **1** **Solution**

(A) $25\pi\ cm^2 s^{-1}$ (B) $30\pi\ cm^2 s^{-1}$ (C) $150\pi\ cm^2 s^{-1}$ (D) $225\pi\ cm^2 s^{-1}$

NESA 2017 Mathematics Extension 1 HSC Examination

16 MX 1 **12 a** The diagram shows a conical soap dispenser of radius 5 cm and height 20 cm. At any time t seconds, the top surface of the soap in the container is a circle of radius r cm and its height is h cm.



The volume of the soap is given by $V = \frac{1}{3}\pi r^2 h$.

(i) Explain why $r = \frac{h}{4}$. **1**

(ii) Show that $\frac{dV}{dh} = \frac{\pi}{16}h^2$. **1**

The dispenser has a leak which causes soap to drip from the dispenser. The area of the circle formed by the top surface of the soap is decreasing at a constant rate of $0.04\ \text{cm}^2\text{s}^{-1}$.

(iii) Show that $\frac{dh}{dt} = \frac{-0.32}{\pi h}$. **2**

(iv) What is the rate of change of the volume of the soap, with respect to time, when $h = 10$? **2**

NESA 2016 Mathematics Extension 1 HSC Examination

Solution

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Year 11: Combinatorics
A1.1 Permutations and combinations

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https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017

Students:

- list and count the number of ways an event can occur
- use the fundamental counting principle (also known as the multiplication principle)
- use factorial notation to describe and determine the number of ways n different items can be arranged in a line or a circle
  - solve problems involving cases where some items are not distinct (excluding arrangements in a circle)
- solve simple problems and prove results using the pigeonhole principle (ACMSM006)
  - understand that if there are n pigeonholes and $n+1$ pigeons to go into them, then at least one pigeonhole must hold 2 or more pigeons
  - generalise to: If n pigeons are sitting in k pigeonholes, where $n > k$, then there is at least one pigeonhole with at least $\frac{n}{k}$ pigeons in it
  - prove the pigeonhole principle
- understand and use permutations to solve problems (ACMSM001)
  - understand and use the notation ${}^nP_r$ and the formula ${}^nP_r = \frac{n!}{(n-r)!}$
- solve problems involving permutations and restrictions with or without repeated objects (ACMSM004)
- understand and use combinations to solve problems (ACMSM007)
  - understand and use the notations $\binom{n}{r}$ and ${}^nC_r$ and the formula ${}^nC_r = \frac{n!}{r!(n-r)!}$ (ACMMM045, ACMSM008)
- solve practical problems involving permutations and combinations, including those involving simple probability situations **AAM**

Reference Sheet

20 MX 1 **8** Out of 10 contestants, six are to be selected for the final round of a competition. Four of those six will be placed 1st, 2nd, 3rd and 4th. **1** Solution

In how many ways can this process be carried out?

A $\frac{10!}{6!4!}$ B $\frac{10!}{6!}$ C $\frac{10!}{4!2!}$ D $\frac{10!}{4!4!}$

NESA 2020 Mathematics Extension 1 HSC Examination

20 MX 1 **12 c** To complete a course, a student must choose and pass exactly three topics. **2** Solution

There are eight topics from which to choose.

Last year 400 students completed the course.

Explain, using the pigeonhole principle, why at least eight students passed exactly the same three topics.

NESA 2020 Mathematics Extension 1 HSC Examination

20 MX 1 **14 a** (i) Use the identity $(1 + x)^{2n} = (1 + x)^n(1 + x)^n$ to show that **2** Solution

$$\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \ldots + \binom{n}{n}^2$$ where n is a positive integer. *P'maths: Binomial Th- next topic*

(ii) A club has $2n$ members, with n women and n men. **2**

A group consisting of an even number (0, 2, 4, …, $2n$) of members is chosen, with the number of men equal to the number of women.

Show, giving reasons, that the number of ways to do this is $\binom{2n}{n}$.

(iii) From the group chosen in part (ii), one of the men and one of the women are selected as leaders. **2**

Show, giving reasons, that the number of ways to choose the even number of people and then the leaders is $1^2\binom{n}{1}^2 + 2^2\binom{n}{2}^2 + \ldots + n^2\binom{n}{n}^2$.

(iv) The process is now reversed so that the leaders, one man and one woman, are chosen first. The rest of the group is then selected, still made up of an equal number of women and men. **2**

By considering this reversed process, and using part (ii), find a simple expression for the sum in part (iii).

NESA 2020 Mathematics Extension 1 HSC Examination

SP MX 1 **7** Each of the students in an athletics team is randomly allocated their own locker from a row of 100 lockers. **1** Solution

What is the smallest number of students in the team that guarantees that two students are allocated consecutive lockers?

A. 26 B. 34 C. 50 D. 51

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **8** A team of 11 students is to be chosen from a group of 18 students. **1** Solution

Among the 18 students are 3 students who are left-handed.

What is the number of possible teams containing at least 1 student who is left-handed?

A. 19 448 B. 30 459 C. 31 824 D. 58 344

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)
NESA 2016 Mathematics Extension 1 HSC Examination

MX SQ 2019 **2** A bag contains 4 white, 4 red and 4 black marbles. Kim takes some marbles out of the bag without looking at them and places them on the table. **1** Solution

What is the smallest number of marbles that must be taken from the bag in order that at least three marbles of the same colour are on the table?

A. 3 B. 5 C. 7 D. 9

NESA Mathematics Extension 1 Sample examination materials (2019)

TG **1** How many numbers greater than 5000 can be formed with the digits 2, 3, 5, 7, 9 if no digit is repeated? Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics

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TG **2** In how many ways can the letters of EERIE be arranged in a line? Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics

TG **3** (a) In how many ways can the numbers 1, 2, 3, 4, 5, 6 be arranged around a circle? **Solution**
(b) How many of these arrangements have at least two even numbers together?

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics

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TG **4** How many arrangements of the letters of the word OLYMPIC are possible if the C and the L are to be together in any order? **Solution**

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics

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TG **5** At the front of a building there are five garage doors. Two of the doors are to be painted red, one is to be painted green, one blue and one orange. **Solution**

10 MX 1 **3a** (a) How many possible arrangements are there for the colours on the doors? **1**

(b) How many possible arrangements are there for the colours on the doors if the two red doors are next to each other? **1**

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics
NESA 2010 Mathematics Extension 1 HSC Examination

TG **6** In horse racing, betting on the trifecta pays on the first three horses in correct order, while the quinella pays on the first two horses in either order. **Solution**

(a) In a 12-horse race, determine the number of possible quinellas.

(b) The Melbourne cup is a 17-horse race.
Determine the number of possible trifectas.

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics

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TG **7** Mr and Mrs Roberts and their four children go to the theatre. **2** **Solution**

07 MX 1 **5b** They are randomly allocated six adjacent seats in a single row.
What is the probability that the four children are allocated seats next to each other?

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics
NESA 2007 Mathematics Extension 1 HSC Examination

TG **8** A four-person team is to be chosen at random from nine women and seven men. **Solution**

04 MX 1 **2e** (i) In how many ways can this team be chosen? **1**

(ii) What is the probability that the team will consist of four women? **1**

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics
NESA 2004 Mathematics Extension 1 HSC Examination

TG **9** Two players *A* and *B* play a series of games against each other to get a prize. In any game, either of the players is equally likely to win. To begin with, the first player who wins a total of 5 games gets the prize. **Solution**

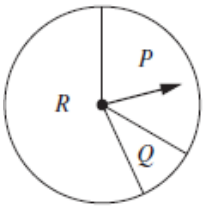
15 MX 1 **14 c** (i) Explain why the probability of player *A* getting the prize in exactly 7 games is $\binom{6}{4}\left(\frac{1}{2}\right)^7$. **1**

(ii) Write an expression for the probability of player *A* getting the prize in at most 7 games. **1**

(iii) Suppose now that the prize is given to the first player to win a total of $(n + 1)$ games, where n is a positive integer. **2**
By considering the probability that *A* gets the prize, prove that
$\binom{n}{n}2^n + \binom{n+1}{n}2^{n-1} + \binom{n+2}{n}2^{n-2} + \ldots + \binom{2n}{n}2^n = 2^{2n}$.

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics
NESA 2015 Mathematics Extension 1 HSC Examination

TG 14 MX 1 **10** **14 b** Two players *A* and *B* play a game that consists of taking turns until a winner is determined. Each turn consists of spinning the arrow on a spinner once. The spinner has three sectors *P*, *Q* and *R*. The probabilities that the arrow stops in sectors *P*, *Q* and *R* are *p*, *q* and *r* respectively.
The rules of the game are as follows:



- If the arrow stops in sector *P*, then the player having the turn wins.
- If the arrow stops in sector *Q*, then the player having the turn loses and the other player wins
- If the arrow stops in sector *R*, then the other player takes a turn.

Player *A* takes the first turn.

(i) Show that the probability of player *A* winning on the first or second turn of the game is $(1 - r)(p + r)$. **2**

(ii) Show that the probability that player *A* eventually wins the game is $\frac{p + r}{1 + r}$. **3**

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics
NESA 2014 Mathematics Extension 1 HSC Examination

Solution

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19 MX 1 **8** In how many ways can all the letters of the word PARALLEL be placed in a line with the three Ls together? **1**

A. $\frac{6!}{2!}$ B. $\frac{6!}{2!3!}$ C. $\frac{8!}{2!}$ D. $\frac{8!}{2!3!}$

NESA 2019 Mathematics Extension 1 HSC Examination

Solution

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18 MX 1 **8** Six men and six women are to be seated at a round table. In how many different ways can they be seated if men and women alternate? **1**

A. 5! 5! B. 5! 6! C. 2! 5! 5! D. 2! 5! 6!

NESA 2018 Mathematics Extension 1 HSC Examination

Solution

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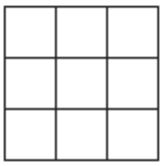
17 MX 1 **10** Three squares are chosen at random from the 3 × 3 grid below, and a cross is placed in each chosen square. What is the probability that all three crosses lie in the same row, column or diagonal? **1**



(A) $\frac{1}{28}$ (B) $\frac{2}{21}$

(C) $\frac{1}{3}$ (D) $\frac{8}{9}$

NESA 2017 Mathematics Extension 1 HSC Examination

Solution

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Year 12: Vectors
V1.2 Further operations with vectors

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Students:

- define, calculate and use the magnitude of a vector in two dimensions and use the notation $\left|\underset{\sim}{u}\right|$ for the magnitude of a vector $\underset{\sim}{u} = x\underset{\sim}{i} + y\underset{\sim}{j}$
  - prove that the magnitude of a vector, $\underset{\sim}{u} = x\underset{\sim}{i} + y\underset{\sim}{j}$, can be found using: $\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$
  - identify the magnitude of a displacement vector $\overrightarrow{AB}$ as being the distance between the points A and B
  - convert a non-zero vector $\underset{\sim}{u}$ into a unit vector $\hat{\underset{\sim}{u}}$ by dividing by its length: $\hat{\underset{\sim}{u}} = \dfrac{\underset{\sim}{u}}{\left|\underset{\sim}{u}\right|}$
- define and use the direction of a vector in two dimensions
- define, calculate and use the scalar (dot) product of two vectors $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$ and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

  AAM
  - apply the scalar product, $\underset{\sim}{u}\cdot\underset{\sim}{v}$, to vectors expressed in component form, where $\underset{\sim}{u}\cdot\underset{\sim}{v} = x_1x_2 + y_1y_2$
  - use the expression for the scalar (dot) product, $\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta$ where θ is the angle between vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ to solve problems
  - demonstrate the equivalence, $\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2$ and use this relationship to solve problems
  - establish and use the formula $\underset{\sim}{v}\cdot\underset{\sim}{v} = \left|\underset{\sim}{v}\right|^2$
  - calculate the angle between two vectors using the scalar (dot) product of two vectors in two dimensions
- examine properties of parallel and perpendicular vectors and determine if two vectors are parallel or perpendicular (ACMSM021)
- define and use the projection of one vector onto another (ACMSM022)
- solve problems involving displacement, force and velocity involving vector concepts in two dimensions (ACMSM023) **AAM**
- prove geometric results and construct proofs involving vectors in two dimensions including to proving that: **AAM**
  - the diagonals of a parallelogram meet at right angles if and only if it is a rhombus (ACMSM039)
  - the midpoints of the sides of a quadrilateral join to form a parallelogram (ACMSM040)
  - the sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides (ACMSM041)

Reference Sheet

20 MX 1 **4** Maria starts at the origin and walks along all of the vector $2\underset{\sim}{i} + 3\underset{\sim}{j}$, then walks along all of the vector $3\underset{\sim}{i} - 2\underset{\sim}{j}$ and finally along all of the vector $4\underset{\sim}{i} - 3\underset{\sim}{j}$. **1** Solution

How far from the origin is she?

A. $\sqrt{77}$ B. $\sqrt{85}$ C. $2\sqrt{13} + \sqrt{5}$ D. $\sqrt{5} + \sqrt{7} + \sqrt{13}$

NESA 2020 Mathematics Extension 1 HSC Examination

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20 MX 1 **11 b** For what value(s) of a are the vectors $\begin{pmatrix} a \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 2a-3 \\ 2 \end{pmatrix}$ perpendicular? **3** Solution

NESA 2020 Mathematics Extension 1 HSC Examination

SP MX 1 **1** What is the angle between the vectors $\begin{pmatrix} 7 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$? **1** Solution

A. $\cos^{-1}(0.6)$ B. $\cos^{-1}(0.06)$ C. $\cos^{-1}(-0.06)$ D. $\cos^{-1}(-0.6)$

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **12 b** A force described by the vector $\underset{\sim}{F} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ newtons is applied to an object lying on a line ℓ which is parallel to the vector $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Solution

(i) Find the component of the force $\underset{\sim}{F}$ in the direction of the line ℓ. **2**

(ii) What is the component of the force $\underset{\sim}{F}$ in the direction perpendicular to the line? **1**

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **12 c** The points A and B are fixed points in a plane and have position vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ respectively. Solution

The point P with position vector $\underset{\sim}{p}$ also lies in the plane and is chosen so that $\angle APB = 90^\circ$.

(i) Explain why $(\underset{\sim}{a} - \underset{\sim}{p}) \cdot (\underset{\sim}{b} - \underset{\sim}{p}) = 0$ **1**

(ii) Let $\underset{\sim}{m} = \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$ denote the position vector of M, the midpoint of A and B. **3**

Using the properties of vectors, show that $|\underset{\sim}{p} - \underset{\sim}{m}|^2$ is independent of $\underset{\sim}{p}$ and find its value. **1**

(iii) What does the result in part (ii) prove about the point P?

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

MX SQ 2019 **15** Points P and A are on the number plane. **4** Solution

The vector is $\overrightarrow{PA}$ is $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

Point B is chosen so that the area of ΔPAB is 10 square units and $|\overrightarrow{PB}| = 4\sqrt{5}$.

Find all possible vectors $\overrightarrow{PB}$.

NESA Mathematics Extension 1 Sample examination materials (2019)

TG 1 Let $\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j}$, $\underset{\sim}{b} = 4\underset{\sim}{i} - 3\underset{\sim}{j}$ and $\underset{\sim}{c} = -2\underset{\sim}{i} - \underset{\sim}{j}$. **Solution**

(a) Calculate $\underset{\sim}{a} \cdot \underset{\sim}{b}$ and $\underset{\sim}{a} \cdot \underset{\sim}{c}$.

(b) Verify that $\underset{\sim}{a} \cdot (\underset{\sim}{b} + \underset{\sim}{c}) = \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{c}$.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG 2 Let $\underset{\sim}{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$. **Solution**

Find the magnitude of $\underset{\sim}{a}$, and find a unit vector in the same direction as $\underset{\sim}{a}$.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG 3 **Solution**

Let *A* and *B* be points with position vectors $\underset{\sim}{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ respectively.

(a) Draw a diagram showing the points *O*, *A* and *B*.

(b) Calculate the angle *AOB*

(i) by finding the tangents of the angles α and β between $\underset{\sim}{a}$ and the unit vector $\underset{\sim}{i}$, and $\underset{\sim}{b}$ and the unit vector $\underset{\sim}{j}$, and using the formula for $\tan(\alpha - \beta)$.

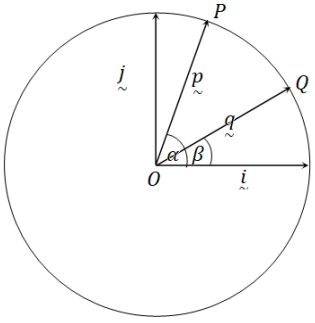
(ii) by using a method based on scalar products.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG 4 Find the angle between the line joining (1, 2) and (3, –5) and the line joining (2, –3) to (1, 4). **Solution**

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG 5 Use vectors and the diagram below of the unit circle to derive the formula for the expansion of $\cos(\alpha - \beta)$ where $0 \le \beta \le \alpha \le \frac{\pi}{2}$. **Solution**



NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG 6 **Solution**

In a triangle *ABC* denote the displacement vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ by $\underset{\sim}{p}$ and $\underset{\sim}{q}$ respectively. Use vectors to prove that, in a triangle *ABC*, $a^2 = b^2 + c^2 - 2bc\cos A$.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG 7 Solution

Let $OABC$ be a kite, with OB as its line of symmetry. Let $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$ and $\overrightarrow{OC} = \underset{\sim}{c}$.

a. Write the vectors $\overrightarrow{AB}$ and $\overrightarrow{CB}$ in terms of $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$.

b. Using the fact that the lengths of AB and CB are equal, write an equation involving scalar products.

c. Use this equation to prove that the diagonals of a kite are perpendicular.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

projectmaths

Year 12: Statistical analysis

C3.2 Normal approximation for the sample proportion

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Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017

Students:

- use appropriate graphs to explore the behaviour of the sample proportion on collected or supplied data **AAM**
  - understand the concept of the sample proportion $\hat{p}$ as a random variable whose value varies between samples (ACMMM174)
- explore the behaviour of the sample proportion using simulated data **AAM**
  - examine the approximate normality of the distribution of $\hat{p}$ for large samples (ACMMM175)
- understand and use the normal approximation to the distribution of the sample proportion and its limitations **AAM**

Reference Sheet

20 MX 1 **12 b** Solution

When a particular biased coin is tossed, the probability of obtaining a head is $\frac{3}{5}$.

This coin is tossed 100 times.

Let X be the random variable representing the number of heads obtained. This random variable will have a binomial distribution.

(i) Find the expected value, $E(X)$. **1**

(ii) By finding the variance, $\text{Var}(X)$, show that the standard deviation of X is approximately 5. **1**

(iii) By using a normal distribution, find the approximate probability that X is between 55 and 65. **1**

NESA 2020 Mathematics Extension 1 HSC Examination

SP MX 1 **12 a** Solution

A recent census showed that 20% of the adults in a city eat out regularly.

(i) A survey of 100 adults in this city is to be conducted to find the proportion who eat out regularly. **2**
Show that the mean and standard deviation for the distribution of sample proportions of such surveys are 0.2 and 0.04 respectivley.

(ii) Use the extract shown from a table giving values of $P(Z < z)$, where z has a standard normal distribution, to estimate the probability that a survey of 100 adults will find that at most 15 of those surveyed eat out regularly. **2**

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

TG **1** State whether the following binomial distributions can or cannot be reasonably approximated by a normal distribution. Solution

Write a brief calculation to justify your conclusion in each case:

(a) Bin(50, 0.2)

(b) Bin(60, 0.1)

(c) Bin(70, 0.01)

(d) Bin(30, 0.7)

(e) Bin(40, 0.9)

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

TG **2** A manufacturer of jam jars knows that 8% of the jars produced are defective. He supplies jars in cartons containing 12 jars. He supplies cartons of jars in crates of 60 cartons. In each case, making clear the distribution that you are using, calculate the probability that: Solution

(a) a carton contains exactly two defective jars.
(b) a carton contains at least one defective jar.
(c) a crate contains between 39 and 44 (inclusive) cartons with at least one defective jar.

Projectmaths has provided this probability table extract:

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
…	…	…	…	…	…	…	…	…	…	…
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

TG **3** A fair coin is tossed 18 times. Use the binomial distribution to find the probability of obtaining 14 Heads. Then use the normal distribution to find the probability of obtaining 14 Heads, and to find the probability of obtaining 14 or more Heads. Solution

Show that the approximation is valid. Projectmaths has provided this probability table extract:

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

TG **4** Suppose that 45% of all HSC students exercise at least four days each week. If a random sample of 50 students is taken, what is the probability that at least 80% of them exercise at least four days per week? Solution

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

TG **5** It is known that 24% of HSC students do not have a driver licence. In a random sample of 16* HSC students, what is the probability that half of them will not have a driver licence? * NESA has 15 … but cannot use 'half of 15' … projectmaths Solution

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

TG **6** A computer simulation is designed to draw random samples of size n from a large dataset. The proportion of the population that exhibits a certain characteristic is $p = 0.25$. **Solution**

If $\hat{p}$ represents the sample proportion exhibiting the characteristic under investigation, find the smallest sample size that should be used so that the standard deviation of $\hat{p}$ is at least 0.01.

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

TG **7** Find the probability of obtaining 4, 5, 6 or 7 Heads when a fair coin is tossed 12 times **Solution**

(a) using the binomial theorem.

(b) using a normal approximation to the binomial distribution.

Projectmaths has provided this probability table extract:

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
…	…	…	…	…	…	…	…	…	…	…
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

TG **8** It is given that 40% of voters support the Stats Party. One hundred and fifty voters are selected at random. Use a suitable approximation to find the probability that more than 55 of the 150 voters support the Stats Party. Projectmaths has provided this prob table extract: **Solution**

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

TG **9** It is estimated that approximately 45% of Australian people will experience a mental illness in their lifetime. If a random sample of 120 mature adults were surveyed, what is the probability of 50 or more having experienced a mental illness? **Solution**

Projectmaths has provided this probability table extract:

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

2020 NESA Sample Paper

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SP MX 1 **1** What is the angle between the vectors $\begin{pmatrix}7\\1\end{pmatrix}$ and $\begin{pmatrix}-1\\1\end{pmatrix}$? **1** Solution

A. $\cos^{-1}(0.6)$ B. $\cos^{-1}(0.06)$ C. $\cos^{-1}(-0.06)$ D. $\cos^{-1}(-0.6)$

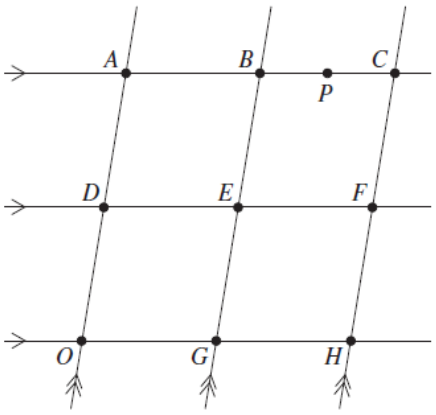
NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **2** The diagram shows a grid of equally spaced lines. **1** Solution

The vector $\overrightarrow{OH} = \underset{\sim}{h}$ and the vector $\overrightarrow{OA} = \underset{\sim}{a}$.

The point P is halfway between B and C.

Which expression represents the vector $\overrightarrow{OP}$?



A. $-\frac{1}{2}\underset{\sim}{a} - \frac{1}{4}\underset{\sim}{h}$ B. $\frac{1}{4}\underset{\sim}{a} - \frac{1}{2}\underset{\sim}{h}$

C. $\underset{\sim}{a} + \frac{1}{4}\underset{\sim}{h}$ D. $\underset{\sim}{a} + \frac{3}{4}\underset{\sim}{h}$

NESA Mathematics Extension 1 Sample HSC Examination (2020)

SP MX 1 **3** Given that $\cos\theta - 2\sin\theta + 2 = 0$, which of the following shows the two possible values for $\tan\frac{\theta}{2}$? **1** Solution

A. -3 or -1 B. -3 or 1 C. -1 or 3 D. 1 or 3

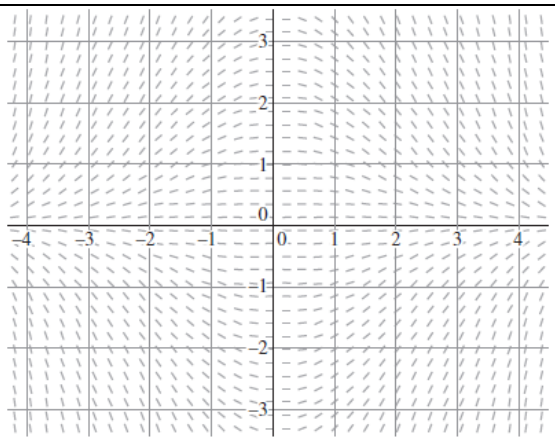
NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **4** What is the derivative of $\tan^{-1}\frac{x}{2}$? **1** Solution

19 MX 1 **3** A. $\frac{1}{2(4+x^2)}$ B. $\frac{1}{4+x^2}$ C. $\frac{2}{4+x^2}$ D. $\frac{4}{4+x^2}$

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)
NESA 2019 Mathematics Extension 1 HSC Examination

SP MX 1 **5** The slope field for a first order differential equation is shown. Which of the following could be the differential equation represented? **1** Solution



A. $\frac{dy}{dx} = \frac{x}{3y}$

B. $\frac{dy}{dx} = -\frac{x}{3y}$

C. $\frac{dy}{dx} = \frac{xy}{3}$

D. $\frac{dy}{dx} = -\frac{xy}{3}$

NESA Mathematics Extension 1 Sample Examination Paper (2020)

SP MX 1 **6** Let $P(x) = qx^3 + rx^2 + rx + q$ where q and r are constants, $q \neq 0$. **1** Solution
One of the zeros of $P(x)$ is -1.
Given that α is a zero of $P(x)$, $\alpha \neq -1$, which of the following is also a zero?

19 MX 1 **7** A. $-\frac{1}{\alpha}$ B. $-\frac{q}{\alpha}$ C. $\frac{1}{\alpha}$ D. $\frac{q}{\alpha}$

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)
NESA 2019 Mathematics Extension 1 HSC Examination

SP MX 1 **7** Each of the students in an athletics team is randomly allocated their own locker from a row of 100 lockers. **1** Solution
What is the smallest number of students in the team that guarantees that two students are allocated consecutive lockers?
A. 26 B. 34 C. 50 D. 51

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **8** A team of 11 students is to be chosen from a group of 18 students. **1** Solution
Among the 18 students are 3 students who are left-handed.
What is the number of possible teams containing at least 1 student who is left-handed?
A. 19 448 B. 30 459 C. 31 824 D. 58 344

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)
NESA 2016 Mathematics Extension 1 HSC Examination

SP MX 1 **9** A stone drops into a pond, creating a circular ripple. **1** Solution
The radius of the ripple increases from 0 cm at a constant rate of 5 cm s^{-1}.
At what rate is the area enclosed within the ripple increasing when the radius is 15 cm?
A. 25π $cm^2 s^{-1}$ B. 30π $cm^2 s^{-1}$ C. 150π $cm^2 s^{-1}$ D. 225π $cm^2 s^{-1}$

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **10** The graph of the function $y = \sin^{-1}(x - 4)$ is transformed by being dilated horizontally with a scale factor of 2 and then translated to the right by 1. **1** Solution
What is the equation of the transformed graph?
A. $y = \sin^{-1}(\frac{x-9}{2})$ B. $y = \sin^{-1}(\frac{x-10}{2})$
C. $y = \sin^{-1}(2x - 6)$ D. $y = \sin^{-1}(2x - 5)$

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **11 a** A particle is fired from the origin O with initial velocity 18 ms^{-1} at an angle 60^o to the horizontal. Solution

The equations of motion are $\frac{d^2x}{dt^2} = 0$ and $\frac{d^2y}{dt^2} = -10$

(i) Show that $x = 9t$. **1**
(ii) Show that $y = 9\sqrt{3}\,t - 5t^2$. **2**
(ii) Hence find the Cartesian equation for the trajectory of the particle. **1**

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

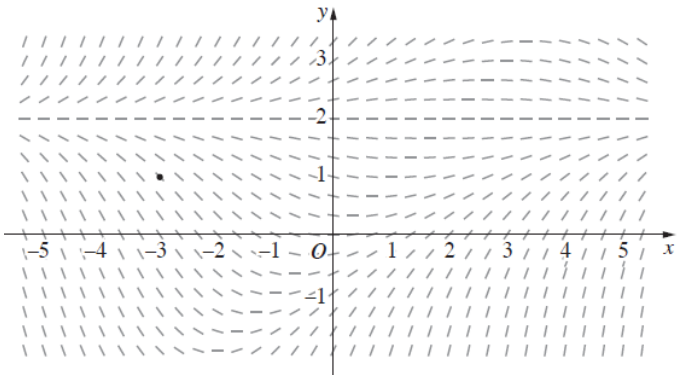
SP MX 1 **11 b** A function $f(x)$ is given by $x^2 + 4x + 7$. Solution
(i) Explain why the domain of the function $f(x)$ must be restricted if $f(x)$ is to have an inverse function. **1**
(ii) Give the equation for $f^{-1}(x)$ if the domain of $f(x)$ is restricted to $x \geq -2$. **2**
(iii) State the domain and range of $f^{-1}(x)$, given the restriction in part (b). **2**
(iv) Sketch the curve $y = f^{-1}(x)$. **2**

NESA Mathematics Extension 1 Sample Examination Paper (2020)

SP MX 1 **11 c** The trajectories of particles in a fluid are described by the differential equation

$$\frac{dy}{dx} = \frac{1}{4}(y - 2)(y - x).$$

The slope field for the differential equation is sketched.



(i) Identify any solutions of the form $y = k$, where k is a constant. **1**

(ii) Draw a sketch of the trajectory of a particle in the fluid which passes through the point $(-3, 1)$ and describe the trajectory as $x \to \pm\infty$. **3**

NESA Mathematics Extension 1 Sample Examination Paper (2020)

Solution

SP MX 1 **12 a** A recent census showed that 20% of the adults in a city eat out regularly.

(i) A survey of 100 adults in this city is to be conducted to find the proportion who eat out regularly. **2**
Show that the mean and standard deviation for the distribution of sample proportions of such surveys are 0.2 and 0.04 respectivley.

(ii) Use the extract shown from a table giving values of $P(Z < z)$, where z has a standard normal distribution, to estimate the probability that a survey of 100 adults will find that at most 15 of those surveyed eat out regularly. **2**

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

Solution

SP MX 1 **12 b** A force described by the vector $\underset{\sim}{F} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ newtons is applied to an object lying on a line ℓ which is parallel to the vector $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

(i) Find the component of the force $\underset{\sim}{F}$ in the direction of the line ℓ. **2**

(ii) What is the component of the force $\underset{\sim}{F}$ in the direction perpendicular to the line? **1**

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

Solution

SP MX 1 **12 c** The points A and B are fixed points in a plane and have position vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ respectively. The point P with position vector $\underset{\sim}{p}$ also lies in the plane and is chosen so that $\angle APB = 90^\circ$. **Solution**

(i) Explain why $(\underset{\sim}{a} - \underset{\sim}{p}) \cdot (\underset{\sim}{b} - \underset{\sim}{p}) = 0$

(ii) Let $\underset{\sim}{m} = \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$ denote the position vector of M, the midpoint of A and B. **1**

Using the properties of vectors, show that $|\underset{\sim}{p} - \underset{\sim}{m}|^2$ is independent of $\underset{\sim}{p}$ and find its value. **3**

(iii) What does the result in part (ii) prove about the point P? **1**

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP 12 **12d 12a** Use mathematical induction to prove that $2^{3n} - 3^n$ is divisible by 5 for $n \geq 1$. **3** **Solution**

NESA Mathematics Extension 1 Sample Examination Paper (2020)
NESA 2012 Mathematics Extension 1 HSC Examination

SP MX 1 **13 a** Use the substitution $x = \sin^2\theta$ to determine $\int_0^{\frac{1}{2}} \sqrt{\frac{x}{1-x}}\, dx$. **3** **Solution**

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **13 b** A device playing a signal given by $x = \sqrt{2}\sin t + \cos t$ produces distortion whenever $|x| \geq 1.5$. For what fraction of the time will the device produce distortion if the signal is played continuously? **4** **Solution**

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **13 c** (i) Prove the trigonometric identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **3** **Solution**

(ii) Hence find expressions for the exact values of the solutions to the equation $8x^3 - 6x = 1$. **4**

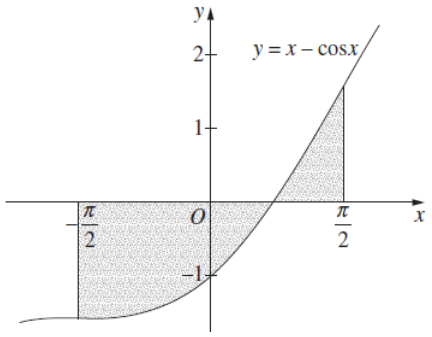
NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **14 a** (i) Sketch the graph of $y = x\cos x$ for $-\pi \leq x \leq \pi$ and hence explain why **3** **Solution**

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x\, dx = 0.$$

(ii) Consider the volume of the solid of revolution produced by rotating about the x-axis the shaded region between the graph of $y = x - \cos x$, the x-axis and the lines $x = -\frac{\pi}{2}$ and $x = -\frac{\pi}{2}$. **3**

Using the results of part (a), or otherwise, find the volume of the solid.



NESA Mathematics Extension 1 Sample Examination Paper (2020)

SP MX 1 **14 b** Solution

The population of foxes on an island is modelled by the logistic equation $\frac{dy}{dt} = y(1 - y)$, where y is the fraction of the isalnd's carrying capacity reached after t years.

At time $t = 0$, the population of foxes is estimated to be one-quarter of the island's carrying capacity.

(i) Use the substitution $y = \frac{1}{1-w}$ to transform the logistic equation to $\frac{dw}{dt} = -w$. **2**

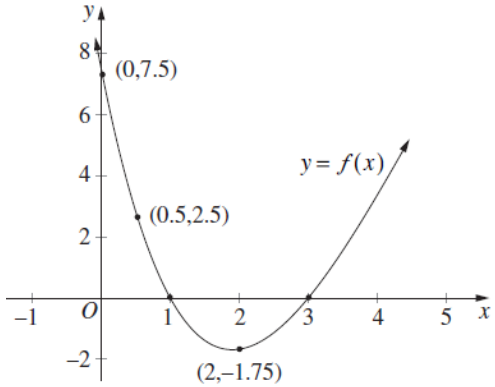
(ii) Using the solution of $\frac{dw}{dt} = -w$, find the solution of the logistic equation for y satisfying the initial conditions. **2**

(iii) How long will it take for the fox population to reach three-quarters of the island's carrying capacity? **2**

NESA Mathematics Extension 1 Sample Examination Paper (2020)

SP MX 1 **14 c** Solution

The diagram is a sketch of the graph of the function $y = f(x)$.



(i) Sketch the graph of $y = \frac{1}{|f(x)|}$. **3**

Your sketch should show any asymptotes and intercepts, together with the location of the points, corresponding to the labelled points on the original sketch.

(ii) How any solutions does the equation $\frac{1}{|f(x)|} = x$ have? **1**

NESA Mathematics Extension 1 Sample Examination Paper (2020)