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MATHEMATICS EXTENSION 1

HSC Exam* Questions by Topic
2019 - 2005

v2020

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Questions by Topic from ...

- **2019 – 2005 HSCs (MX1: Mathematics Extension 1, M: Mathematics)**
- **NESA Sample examination questions [SQ]**
- **NESA Topic Guidance [TG]**

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Year 11: Functions
F1.3 Inverse functions

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Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017

Students:

- define the inverse relation of a function $y = f(x)$ to be the relation obtained by reversing all the ordered pairs of the function
- examine and use the reflection property of the graph of a function and the graph of its inverse (ACMSM096)
  - understand why the graph of the inverse relation is obtained by reflecting the graph of the function in the line $y = x$
  - using the fact that this reflection exchanges horizontal and vertical lines, recognise that the horizontal line test can be used to determine whether the inverse relation of a function is again a function
- write the rule or rules for the inverse relation by exchanging x and y in the function rules, including any restrictions, and solve for y, if possible
- when the inverse relation is a function, use the notation $f^{-1}(x)$ and identify the relationships between the domains and ranges of $f(x)$ and $f^{-1}(x)$
- when the inverse relation is not a function, restrict the domain to obtain new functions that are one-to-one, and compare the effectiveness of different restrictions
- solve problems based on the relationship between a function and its inverse function using algebraic or graphical techniques **AAM**

Reference Sheet

SQ **13** A function $f(x)$ is given by $x^2 + 4x + 7$. Solution

(a) Explain why the domain of the function $f(x)$ must be restricted if $f(x)$ is to have an inverse function. **1**

(b) Give the equation for $f^{-1}(x)$ if the domain of $f(x)$ is restricted to $x \geq -2$. **2**

(c) State the domain and range of $f^{-1}(x)$, given the restriction in part (b). **2**

(d) Sketch the curve $y = f^{-1}(x)$ in the space provided. **2**

NESA Mathematics Extension 1 Sample examination materials

TG **1** For each function, state the domain and range of $f^{-1}(x)$ and sketch the graphs of $f(x)$ and $f^{-1}(x)$ on the same axes: Solution

(a) $f(x) = x^3$

(b) $f(x) = 1 - 3x$

(c) $f(x) = x^3 + 5$

(d) $f(x) = \sqrt{x}$, for $x > 0$

(e) $f(x) = (x - 1)^2 - 6$ for $x \geq 1$

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

TG **2** Functions f and g are defined by $f(x) = 4x + 5$ and $g(x) = 3 - 2x$. Solution

Find the inverse of the composite function $f \circ g$.

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

TG **3** The function f is defined by $f(x) = x^2 - 2x + 7$ with domain $x \leq k$. Solution
Given that f is a one-to-one function, find the greatest possible value of k and find the inverse function f^{-1}.

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

TG **4** Graph the inverse of $f(x) = \sqrt{x+1}$. [Solution]

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

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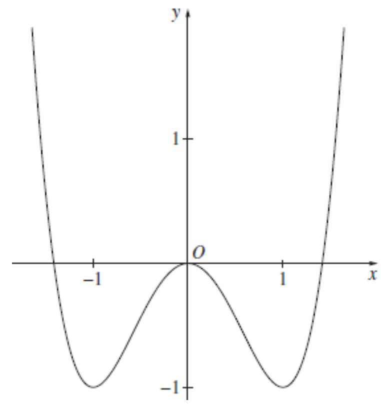
19 MX 1 **10** The function $f(x) = -\sqrt{1+\sqrt{1+x}}$ has inverse $f^{-1}(x)$. **1** [Solution]

The graph of $y = f^{-1}(x)$ forms part of the curve $y = x^4 - 2x^2$.

The diagram shows the curve $y = x^4 - 2x^2$.



How many points do the graphs of $y = f(x)$ and $y = f^{-1}(x)$ have in common?

A. 1 B. 2

C. 3 D. 4

NESA 2019 Mathematics Extension 1 HSC Examination

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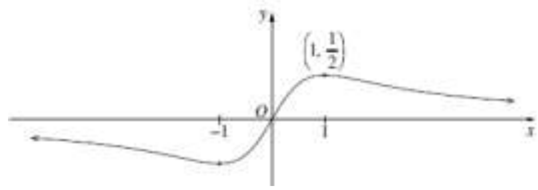
18 MX 1 **13 b** The diagram shows the graph $y = \dfrac{x}{x^2+1}$, for all real x. [Solution]



Consider the function $y = \dfrac{x}{x^2+1}$, for $x \geq 1$.

The function $f(x)$ has an inverse. (Do NOT prove this.)

(i) State the domain and range of $y = f^{-1}(x)$. **2**

(ii) Sketch the graph of $y = f^{-1}(x)$. **1**

(iii) Find an expression for $f^{-1}(x)$. **3**

NESA 2018 Mathematics Extension 1 HSC Examination

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16 MX 1 **11 a** Find the inverse of the function $y = x^3 - 2$. **2** [Solution]

NESA 2016 Mathematics Extension 1 HSC Examination

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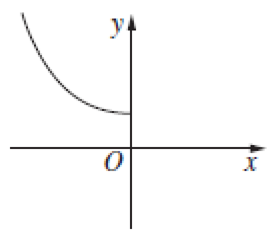
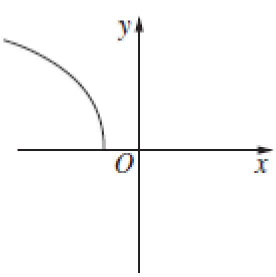
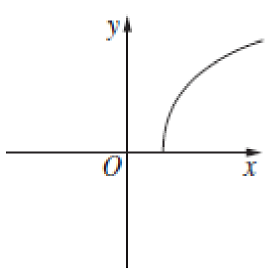
13 MX 1 **2** The diagram shows the graph $y = f(x)$. **1** [Solution]



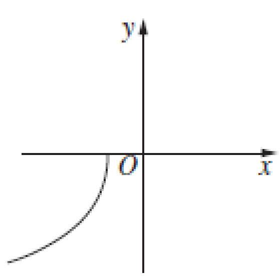
Which diagram shows the graph of $y = f^{-1}(x)$?



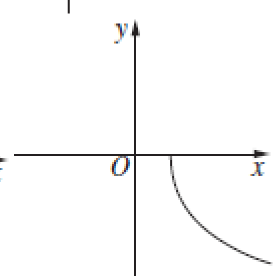
(A)



(B)



(C)



(D)

NESA 2013 Mathematics Extension 1 HSC Examination

12 MX 1 **12 b** Solution

Let $f(x) = \sqrt{4x-3}$

(i) Find the domain of $f(x)$. **1**

(ii) Find an expression for the inverse function $f^{-1}(x)$. **2**

(iii) Find the points where the graphs $y = f(x)$ and $y = x$ intersect. **1**

(iv) On the same set of axes, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ showing the information found in part (iii). **2**

NESA 2012 Mathematics Extension 1 HSC Examination

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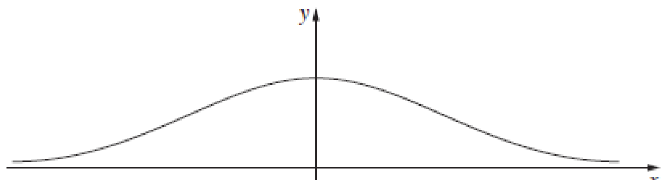
10 MX 1 **3b** Solution

Let $f(x) = e^{-x^2}$. The diagram shows the graph $y = f(x)$.



(i) The graph has two points of inflexion. Find the x coordinate of these points. **3**

(ii) Explain why the domain of $f(x)$ must be restricted if $f(x)$ is to have an inverse function. **1**

(iii) Find a formula for $f^{-1}(x)$ if the domain of f(x) is restricted to $x \geq 0$. **2**

(iv) State the domain of $f^{-1}(x)$. **1**

(v) Sketch the curve $y = f^{-1}(x)$. **1**

NESA 2010 Mathematics Extension 1 HSC Examination

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09 MX 1 **3a** Solution

Let $f(x) = \dfrac{3+e^{2x}}{4}$

(i) Find the range of $f(x)$ **1**

(ii) Find the inverse function $f^{-1}(x)$. **2**

NESA 2009 Mathematics Extension 1 HSC Examination

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08 MX 1 **5a** Solution

Let $f(x) = x - \frac{1}{2}x^2$ for $x \leq 1$. This function has an inverse, $f^{-1}(x)$.

(i) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes. (Use the same scale on both axes.) **2**

(ii) Find an expression for $f^{-1}(x)$. **3**

(iii) Evaluate $f^{-1}(\frac{3}{8})$. **1**

NESA 2008 Mathematics Extension 1 HSC Examination

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07 MX 1 **6b** Solution

Consider the function $f(x) = e^x - e^{-x}$.

(i) Show that $f(x)$ is increasing for all values of x. **1**

(ii) Show that the inverse function is given by **3**

$$f^{-1}(x) = \log_e\left(\frac{x+\sqrt{x^2+4}}{2}\right)$$

(iii) Hence, or otherwise, solve $e^x - e^{-x} = 5$. **1**
Give your answer correct to two decimal places.

NESA 2007 Mathematics Extension 1 HSC Examination

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06 MX 1 **5b** Let $f(x) = \log_e(1 + e^x)$ for all x. Show that $f(x)$ has an inverse. **2** Solution

NESA 2006 Mathematics Extension 1 HSC Examination

Year 11: Calculus
C1.3 Related rates of change

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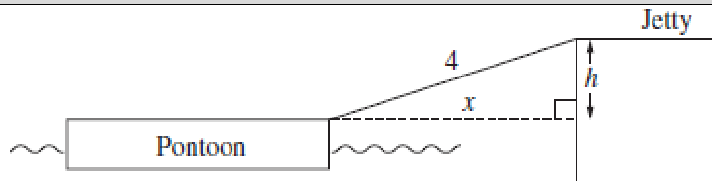
Students:

- solve problems involving related rates of change as instances of the chain rule (ACMSM129) **AAM**
- develop models of contexts where a rate of change of a function can be expressed as a rate of change of a composition of two functions, and to which the chain rule can be applied

Reference Sheet

TG **1** **04 MX 1** **3c**

A ferry wharf consists of a floating pontoon linked to a jetty by a 4 metre long walkway. Let h metres be the difference in height between the top of the pontoon and the top of the jetty and



let x metres be the horizontal distance between the pontoon and the jetty.

(i) Find an expression for x in terms of h. **1**

(ii) When the top of the pontoon is 1 metre lower than the top of the jetty, the tide is rising at a rate of 0.3 metres per hour. **3**
At what rate is the pontoon moving away from the jetty?

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus
NESA 2004 Mathematics Extension 1 HSC Examination

Solution

TG **2**

A spherical balloon is being deflated so that the radius decreases at a constant rate of 10 mm per second.
Calculate the rate of change of volume when the radius of the balloon is 100 mm.

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus

Solution

TG **3**

A spherical bubble is expanding so that its volume increases at the constant rate of 70 mm^3 per second.
What is the rate of increase of its surface area when the radius is 10 mm?

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus

Solution

TG **4**

A hot air balloon is at a constant height of 160 metres above the ground, and moving parallel to the ground, at a speed of 20 metres per minute. Find the rate at which the balloon is moving away from an observer on the ground at the time when the distance from the observer to the balloon is 400 metres.

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus

Solution

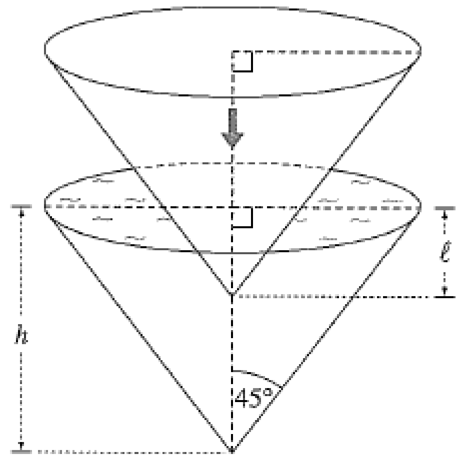
TG **5**

11 MX 1 **7a**

Solution

The diagram shows two identical circular cones with a common vertical axis. Each cone has height h cm and semi-vertical angle 45°.

The lower cone is completely filled with water. The upper cone is lowered vertically into the water as shown in the diagram. The rate at which it its lowered is given by $\frac{d\ell}{dt} = 10$, where ℓ cm is the distance the upper cone has descended into the water after t seconds. As the upper cone is lowered, water spills from the lower cone. The volume of water remaining in the lower cone at time t is V cm^3.



Not to scale

(i) Show that $V = \frac{\pi}{3}\left(h^3 - \ell^3\right)$. **1**

(ii) Find the rate at which V is changing with respect to time when $\ell = 2$. **2**

(iii) Find the rate at which V is changing with respect to time when the lower cone has lost $\frac{1}{8}$ of its water. Give your answer in terms of h. **2**

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus
NESA 2011 Mathematics Extension 1 HSC Examination

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19 MX 1 **12 a**

Distance A is inversely proportional to distance B, such that $A = \frac{9}{B}$, where A and B are measured in metres. The two distances vary with respect to time. Distance B is increasing at a rate of 0.2 ms^{-1}. What is the value of $\frac{dA}{dt}$ when $A = 12$? **3**

Solution

NESA 2019 Mathematics Extension 1 HSC Examination

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17 MX 1 **8**

A stone drops into a pond, creating a circular ripple. The radius of the ripple increases from 0 cm, at a constant rate of 5 cm s^{-1}. At what rate is the area of enclosed within the ripple increasing when the radius is 15 cm? **1**

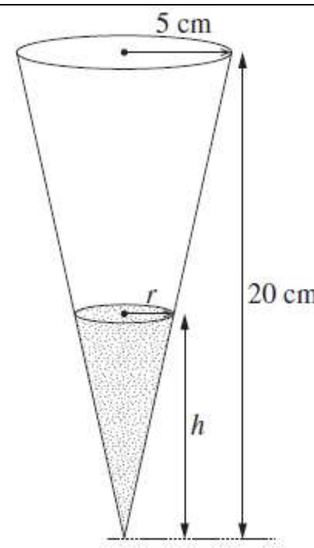
Solution

(A) 25π cm^{2}s^{-1} (B) 30π cm^{2}s^{-1} (C) 150π cm^{2}s^{-1} (D) 225π cm^{2}s^{-1}

NESA 2017 Mathematics Extension 1 HSC Examination

16 MX 1 **12 a**

The diagram shows a conical soap dispenser of radius 5 cm and height 20 cm. At any time t seconds, the top surface of the soap in the container is a circle of radius r cm and its height is h cm.



Solution

The volume of the soap is given by $v = \frac{1}{3}\pi r^2 h$.

(i) Explain why $r = \frac{h}{4}$. **1**

(ii) Show that $\frac{dv}{dh} = \frac{\pi}{16}h^2$. **1**

The dispenser has a leak which causes soap to drip from the dispenser. The area of the circle formed by the top surface of the soap is decreasing at a constant rate of $0.04\ \text{cm}^2\text{s}^{-1}$.

(iii) Show that $\frac{dh}{dt} = \frac{-0.32}{\pi h}$. **2**

(iv) What is the rate of change of the volume of the soap, with respect to time, when $h = 10$? **2**

NESA 2016 Mathematics Extension 1 HSC Examination

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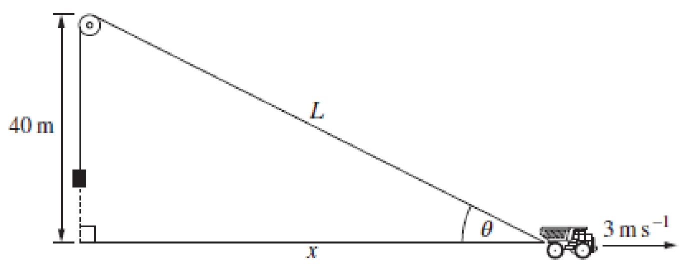
14 MX 1 **13 b**

One end of a rope is attached to a truck and the other end to a weight. The rope passes over a small wheel located at a vertical distance of 40 m above the point where the rope is attached to the truck. The distance from the truck to the small wheel is L m and the horizontal distance between them is x m. The rope makes an angle θ with the horizontal at the point where it is attached to the truck. The truck moves left to right at a constant speed of 3 ms^{-1}, as shown in the diagram.



Solution

(i) Using Pythagoras' Theorem, or otherwise, show that $\frac{dL}{dx} = \cos\theta$. **2**

(ii) Show that $\frac{dL}{dt} = 3\cos\theta$. **1**

NESA 2014 Mathematics Extension 1 HSC Examination

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13 MX 1 **13 a**

A spherical raindrop of radius r metres loses water through evaporation at a rate that depends on its surface area.

Solution

The rate of change of the volume V of the raindrop is given by $\frac{dV}{dt} = -10^{-4}A$,

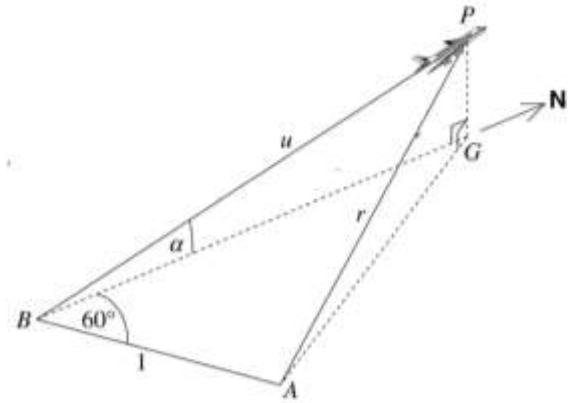
where t is time in seconds and A is the surface area of the raindrop.
The surface area and the volume of the raindrop are given by $A = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$ respectively.

(i) Show that $\frac{dr}{dt}$ is constant. **1**

(ii) How long does it take for a raindrop of volume $10^{-6}\ \text{m}^3$ to completely evaporate? **2**

NESA 2013 Mathematics Extension 1 HSC Examination

12 MX 1 **14 c** A plane P takes off from a point B. It flies due north at a constant angle α to the horizontal. An observer is located at A, 1 km from B, at a bearing 060° from B. Let u km be the distance from B to the plane and let r km be the distance from the observer to the plane. The point G is on the ground directly below the plane. [Solution]



(i) Show that $r = \sqrt{1 + u^2 - u\cos\alpha}$. **3**

(ii) The plane is travelling at a constant speed of 360 km/h. At what rate, in terms of α, is the distance of the plane from the observer changing 5 minutes after take-off? **2**

NESA 2012 Mathematics Extension 1 HSC Examination

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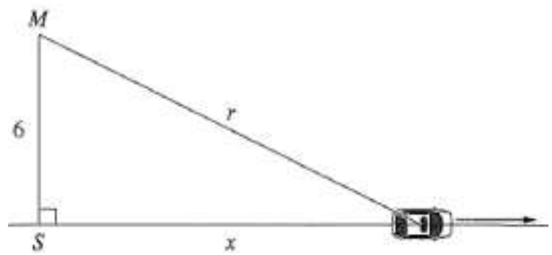
10 MX 1 **2d** A radio transmitter M is situated 6 km from a straight road. The closest point on the road to the transmitter is S. A car travelling away from S along the road at a speed of 100 km h^{-1}. The distance from the car to S is x km and from the car to M is r km. **3** [Solution]



Find an expression in terms of x for $\frac{dr}{dt}$, where t is time in hours.

NESA 2010 Mathematics Extension 1 HSC Examination

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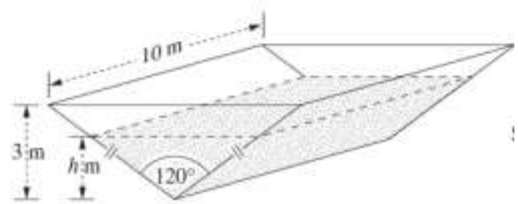
09 MX 1 **5b** The cross-section of a 10 metre long tank is an isosceles triangle, as shown in the diagram. The top of the tank is horizontal. When the tank is full, the depth of water is 3 m. The depth of water at time t days is h metres. [Solution]



(i) Find the volume, V, of water in the tank when the depth of water is h metres. **1**

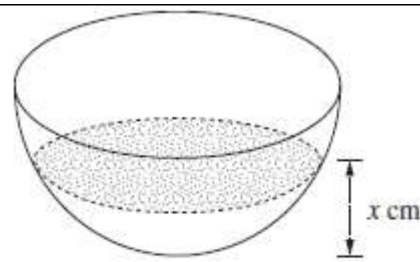
(ii) Show that the area, A, of the top surface of the water is given by $A = 20\sqrt{3}\,h$ **1**

(iii) The rate of evaporation of the water is given by $\frac{dV}{dt} = -kA$, where k is a positive constant. Find the rate at which the depth of water is changing at time t. **2**

(iv)* It takes 100 days for the depth to fall from 3 m to 2 m. Find the time taken for the depth to fall from 2 m to 1 m. **1**
(* involves integration (y12))

NESA 2009 Mathematics Extension 1 HSC Examination

06 MX 1 **5c** A hemispherical bowl of radius r cm is initially empty. Water is poured into it at a constant rate of k cm^3 per minute. When the depth of water in the bowl is x cm, the volume, V cm^3, of water in the bowl is given by $V = \frac{\pi}{3}x^2(3r - x)$.



[Solution](#)

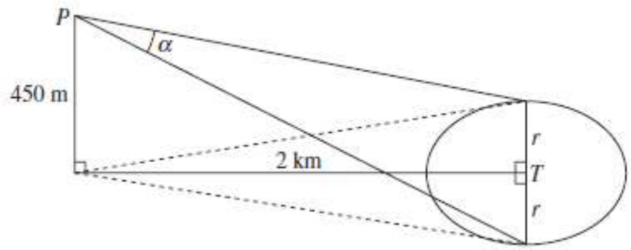
(Do NOT prove this.)

(i) Show that $\frac{dx}{dt} = \frac{k}{\pi x(2r - x)}$. **2**

(ii) Hence, or otherwise, show that it takes 3.5 times as long to fill the bowl to the point where $x = \frac{2}{3}r$ as it does to fill the bowl to the point where $x = \frac{1}{3}r$ **2**

NESA 2006 Mathematics Extension 1 HSC Examination

05 MX 1 **7a** An oil tanker at T is leaking oil which forms a circular oil slick. An observer is measuring the oil slick from a position P, 450 metres above sea level and 2 kilometres horizontally from the centre of the oil slick.



[Solution](#)

(i) At a certain time the observer measures the angle, α, subtended by the diameter of the oil slick, to be 0.1 radians. What is the radius, r, at this time? **2**

(ii) At this time, $\frac{d\alpha}{dt} = 0.02$ radians per hour. Find the rate at which the radius of the oil slick is growing. **2**

NESA 2005 Mathematics Extension 1 HSC Examination

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Year 12: Proof

P1 Proof by mathematical induction

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Students:

- understand the nature of inductive proof, including the 'initial statement' and the inductive step (ACMSM064)
- prove results using mathematical induction
  - prove results for sums, for example $1 + 4 + 9 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for any positive integer n (ACMSM065)
  - prove divisibility results, for example $3^{2n} - 1$ is divisible by 8 for any positive integer n (ACMSM066)
- identify errors in false 'proofs by induction', such as cases where only one of the required two steps of a proof by induction is true, and understand that this means that the statement has not been proved
- recognise situations where proof by mathematical induction is not appropriate

Reference Sheet

SQ 12 MX 1	**12 12a**	Use mathematical induction to prove that $2^{3n} - 3^n$ is divisible by 5 for $n \ge 1$. NESA Mathematics Extension 1 Sample examination materials NESA 2012 Mathematics Extension 1 HSC Examination	**3**	**Solution**
TG 08 MX 1	**1 3b**	Use mathematical induction to prove that, for all integers $n \geq 1$, $1 \times 3 + 2 \times 4 + 3 \times 5 + \ldots + n(n + 2) = \frac{n}{6}(n + 1)(2n + 7)$. NESA Mathematics Extension 1 Year 12 Topic Guide: Proof NESA 2008 Mathematics Extension 1 HSC Examination	**3**	**Solution**
TG	**2**	Prove by mathematical induction that $3^{2n+4} - 2^{2n}$ is a multiple of 5, for all positive integers n. NESA Mathematics Extension 1 Year 12 Topic Guide: Proof		**Solution**
TG	**3**	Prove by mathematical induction that $n^3 + 2n$ is divisible by 3 for all positive integers n. NESA Mathematics Extension 1 Year 12 Topic Guide: Proof		**Solution**
TG	**4**	Show that the inductive step can be proven for the false proposition that $1 + 2 + 3 + \ldots + n = \frac{1}{2}(n - 1)(n + 2)$ for integers $n \ge 1$, but the initial case does not hold true. NESA Mathematics Extension 1 Year 12 Topic Guide: Proof		**Solution**
19 MX 1	**14 a**	Prove by mathematical induction that, for all integers $n \ge 1$, $1(1!) + 2(2!) + 3(3!) + \ldots + n(n!) = (n + 1)! - 1$ NESA 2019 Mathematics Extension 1 HSC Examination	**3**	**Solution**
18 MX 1	**13 a**	Prove by mathematical induction that, for $n \ge 1$, $2 - 6 + 18 - 54 + \ldots + 2(-3)^{n-1} = \frac{1-(-3)^n}{2}$ NESA 2018 Mathematics Extension 1 HSC Examination	**3**	**Solution**
17 MX 1	**14 a**	Prove by mathematical induction that $8^{2n+1} + 6^{2n-1}$ is divisible by 7, for any integer $n \ge 1$. NESA 2017 Mathematics Extension 1 HSC Examination	**3**	**Solution**

16 MX 1	**14 a**	(i) Show that $4n^3 + 18n^2 + 23n + 9$ can be written as $(n + 1)(4n^2 + 14n + 9)$. (ii) Using the results in part (i), or otherwise, prove by mathematical induction that, for $n \geq 1$, $1 \times 3 + 3 \times 5 + 5 \times 7 + \ldots + (2n - 1)(2n + 1) = \frac{1}{3}n(4n^2 + 6n - 1)$. NESA 2016 Mathematics Extension 1 HSC Examination	**1** **3**	Solution
15 MX 1	**13 c**	Prove by mathematical induction that for all integers $n \geq 1$, $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}$. NESA 2015 Mathematics Extension 1 HSC Examination	**3**	Solution
14 MX 1	**13 a**	Use mathematical induction to prove that $2^n + (-1)^{n+1}$ is divisible by 3 for all integers $n \geq 1$. NESA 2014 Mathematics Extension 1 HSC Examination	**3**	Solution
11 MX 1	**6a**	Use mathematical induction to prove that, for $n \geq 1$, $1 \times 5 + 2 \times 6 + 3 \times 7 + \ldots + n(n + 4) = \frac{1}{6}n(n + 1)(2n + 13)$. NESA 2011 Mathematics Extension 1 HSC Examination	**3**	Solution
10 MX 1	**7a**	Prove by induction that $47^n + 53 \times 147^{n-1}$ is divisible by 100 for all integers $n \geq 1$. NESA 2010 Mathematics Extension 1 HSC Examination	**3**	Solution
07 MX 1	**4b**	Use mathematical induction to prove that, $7^{2n-1} + 5$ is divisible by 12, for all integers $n \geq 1$. NESA 2007 Mathematics Extension 1 HSC Examination	**3**	Solution
06 MX 1	**5d**	(i) Use the fact that $\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$ to show that $1 + \tan n\theta\tan(n+1)\theta = \cot\theta\,(\tan(n+1)\theta - \tan n\theta)$ (ii) Use mathematical induction to prove that, for all integers $n \geq 1$, $\tan\theta\tan 2\theta + \tan 2\theta\tan 3\theta + \ldots + \tan n\theta\tan(n+1)\theta$ $= -(n+1) + \cot\theta\tan(n+1)\theta$ NESA 2006 Mathematics Extension 1 HSC Examination	**1** **3**	Solution

projectmaths

Year 12: Calculus
C3.1 Further area and volume of solids

Back

Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017

Students:

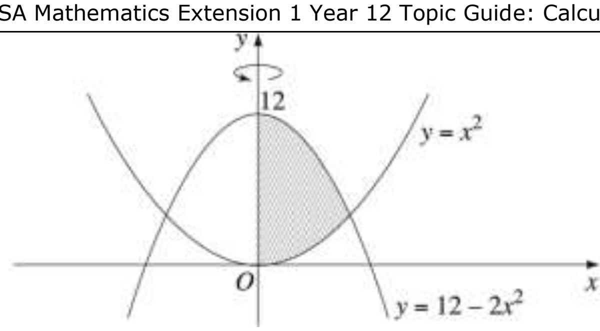
- calculate area of regions between curves determined by functions (ACMSM124)
- sketch, with and without the use of technology, the graph of a solid of revolution whose boundary is formed by rotating an arc of a function about the x-axis or y-axis **AAM**
- calculate the volume of a solid of revolution formed by rotating a region in the plane about the x-axis or y-axis, with and without the use of technology (ACMSM125) **AAM**
- determine the volumes of solids of revolution that are formed by rotating the region between two curves about either the x-axis or y-axis in both real-life and abstract contexts **AAM**

Reference Sheet

TG **1** Sketch the region bounded by the curve $y = x^2$ and the lines $y = 4$ and $y = 9$. Evaluate the area of this region. **Solution**

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **2** The graphs of the curves $y = x^2$ and $y = 12 - 2x^2$ are shown in the diagram. **Solution**



(a) Find the points of intersection of the two curves.

(b) The shaded region between the curves and the y-axis is rotated about the y-axis. By splitting the shaded region into two parts, or otherwise, find the volume of the solid formed.

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **3** The region bounded by the curve $y = (x - 1)(3 - x)$ and the x-axis is rotated about the line $x = 3$ to form a solid. **Solution**

When the region is rotated, the horizontal line segment at height y sweeps out an annulus.

(a) Find the area of the annulus as a function of y.

(b) Find the volume of the solid.

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **4** The region enclosed by the curve $y = 4\sqrt{x}$ and the x-axis between $x = 0$ and $x = 4$ is rotated about the x-axis.
Find the volume of the solid of revolution. **Solution**

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **5** A curved funnel has a shape formed by rotating part of the parabola $y = 2\sqrt{x}$ about the y-axis, where x and y are given in cm.
The funnel is 4 cm deep.
Find the volume of liquid which the funnel will hold if it is sealed at the bottom. **Solution**

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **6** (a) Sketch the region bounded by the curve $y = \sin x + \cos x$ and the coordinate axes in the first quadrant, taking the upper limit of x as $\frac{3\pi}{4}$. Show the intercepts on the axes, and calculate the area of the region. **Solution**

(b) Find the volume of the solid formed if the region is rotated about the x-axis to form a solid of revolution.

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

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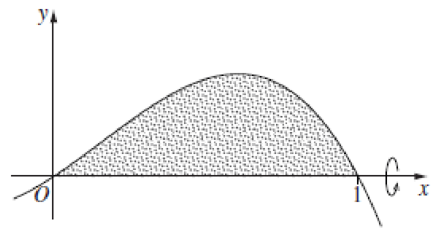
19 M **13 d** The diagram shows the region bounded by the curve $y = x - x^3$, and the x-axis between $x = 0$ and $x = 1$.
The region is rotated about the x-axis to form a solid.
Find the exact value of the volume of the solid formed. **3** **Solution**



NESA 2019 Mathematics HSC Examination

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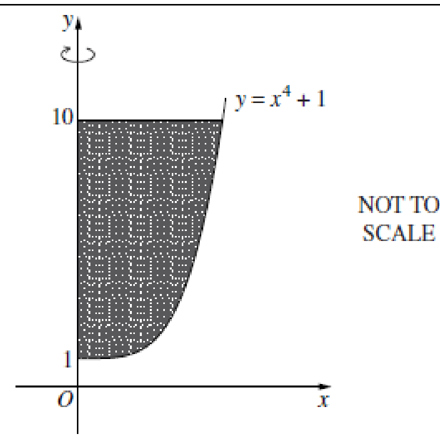
18 M **14 b** The shaded region shown in the diagram is bounded by the curve $y = x^4 + 1$, then y-axis and the line $y = 10$.
Find the volume of the solid of revolution formed when the shaded region is rotated about the y-axis.
. **3** **Solution**



NESA 2018 Mathematics HSC Examination

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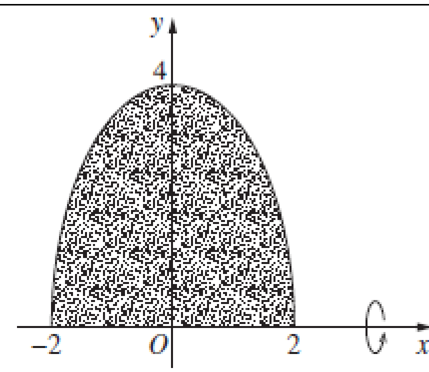
17 M **12 b** The diagram shows the region bounded by $y = \sqrt{16 - 4x^2}$ and the x-axis.
The region is rotated about the x-axis to form a solid.
Find the exact volume of the solid formed. **3** **Solution**

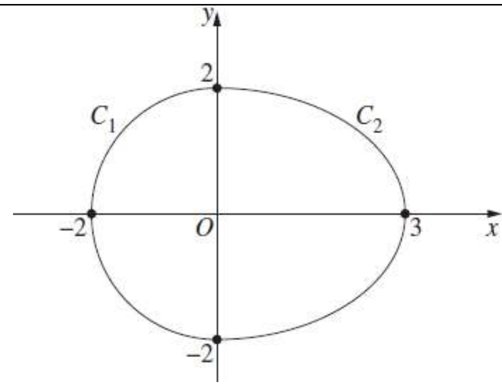


NESA 2017 Mathematics HSC Examination

16 M **15 a**

The diagram shows two curves C_1 and C_2. The curve C_1 is the semicircle $x^2 + y^2 = 4$, $-2 \le x \le 2$. The curve C_2 has equation $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$, $0 \le x \le 3$.

An egg is modelled by rotating the curves about the x-axis to form a solid of revolution.
Find the exact value of the volume of the solid of revolution.



NESA 2016 Mathematics HSC Examination

4 [Solution]

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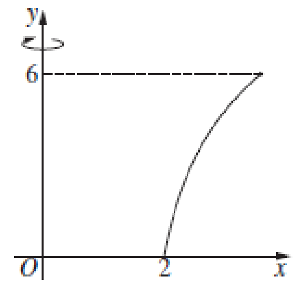
15 M **16 b**

A bowl is formed by rotating the curve $y = 8\log_e(x - 1)$ about the y-axis for $0 \le y \le 6$.

Find the volume of the bowl.
Give your answer correct to 1 decimal place.

Not to scale



NESA 2015 Mathematics HSC Examination

3 [Solution]

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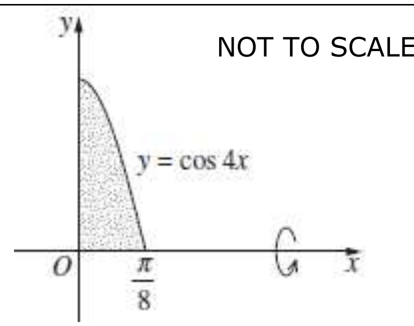
14 MX 1 **12 b**

The region bounded by $y = \cos 4x$ and the x-axis, between $x = 0$ and $x = \dfrac{\pi}{8}$, is rotated about the x-axis to form a solid.
Find the volume of the solid.



NESA 2014 Mathematics Extension 1 HSC Examination

3 [Solution]

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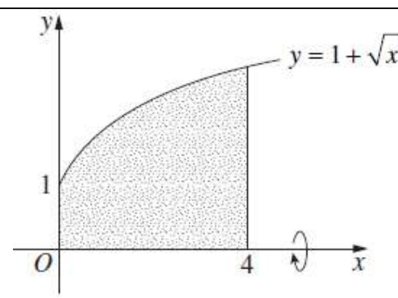
14 M **14 c**

The region bounded by the curve $y = 1 + \sqrt{x}$ and the x-axis between $x = 0$ and $x = 4$ is rotated about the x–axis to form a solid.
Find the volume of the solid.



NESA 2014 Mathematics HSC Examination

3 [Solution]

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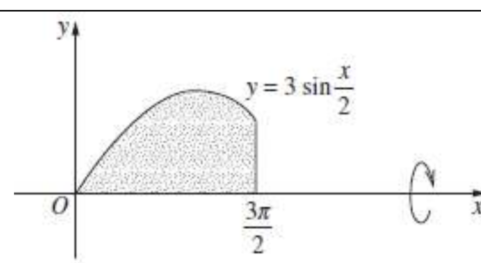
13 MX 1 **12 b**

The region bounded by the graph $y = 3\sin\dfrac{x}{2}$ and the x-axis between $x = 0$ and $x = \dfrac{3\pi}{2}$ is rotated about the x-axis to form a solid.

Find the exact volume of the solid.



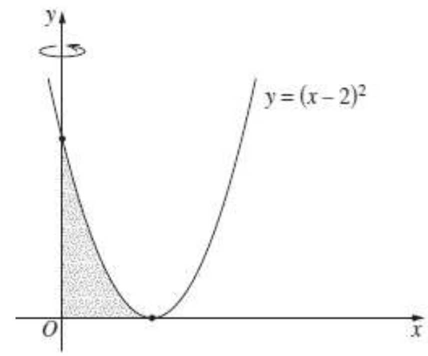
NESA 2013 Mathematics Extension 1 HSC Examination

3 [Solution]

13 M **15 b**

The region bounded by the x-axis, the y-axis and the parabola $y = (x - 2)^2$ is rotated about the y-axis to form a solid.

Find the volume of the solid. **4** [Solution]



NESA 2013 Mathematics HSC Examination

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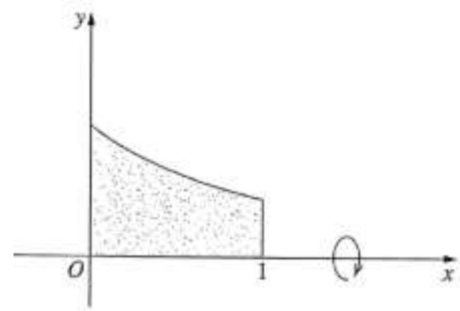
12 M **14 b**

The diagram shows the region bounded by $y = \dfrac{3}{(x+2)^2}$, the x-axis, the y-axis, and the line $x = 1$.
The region is rotated about the x-axis to form a solid.
Find the volume of the solid. **3** [Solution]



NESA 2012 Mathematics HSC Examination

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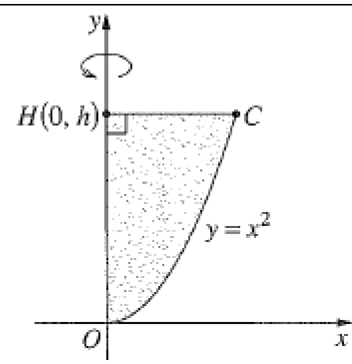
11 M **8b**

The diagram shows the region enclosed by the parabola $y = x^2$, the y-axis and the line $y = h$, where $h > 0$. This region is rotated about the y-axis to form a solid called a paraboloid. The point C is the intersection of $y = x^2$ and $y = h$. The point H has coordinates $(0, h)$. [Solution]

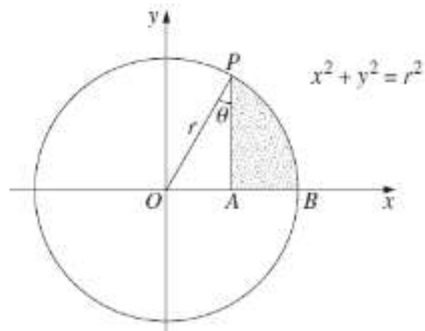
(i) Find the exact volume of the paraboloid in terms of h. **2**

(ii) A cylinder has radius HC and height h. What is the ratio of the volume of the paraboloid to the volume of the cylinder? **1**



NESA 2011 Mathematics HSC Examination

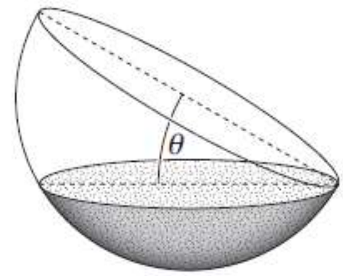
10 M **10 b** The circle $x^2 + y^2 = r^2$ has radius r and centre O. The circle meets the positive x-axis at B. The point A is on the interval OB. A vertical line through A meets the circle at P. Let $\theta = \angle OPA$. Solution



(i) The shaded region bounded by the arc PB and the intervals AB and AP is rotated about the x-axis. Show that the volume, V, formed is given by **3**

$$V = \frac{\pi r^3}{3}(2 - 3\sin\theta + \sin^3\theta).$$

(ii) A container is in the shape of a hemisphere of radius r metres. The container is initially horizontal and full of water. The container is then tilted at an angle of θ to the horizontal so that some water spills out.



(1) Find θ so that the depth of water remaining is one half of the original depth. **1**

(2) What fraction of the original volume is left in the container? **2**

NESA 2010 Mathematics HSC Examination

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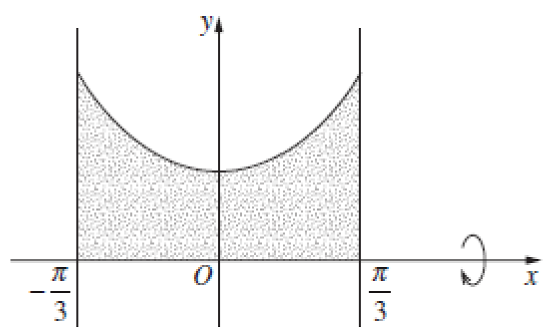
09 M **6a** The diagram shows the region bounded by the curve $y = \sec x$, the lines $x = \frac{\pi}{3}$ and $x = -\frac{\pi}{3}$, and the x-axis. Solution

The region is rotated about the x-axis. Find the volume of the solid of revolution formed. **2**



NESA 2009 Mathematics HSC Examination

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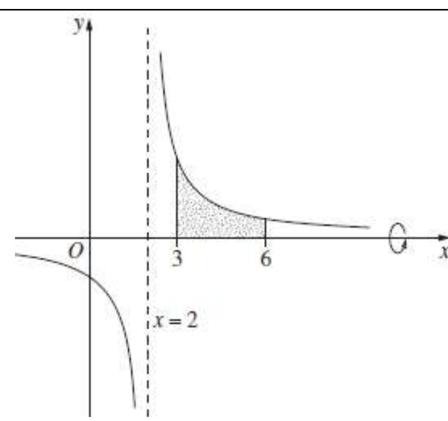
08 M **6c** The graph of $y = \frac{5}{x-2}$ is shown. The shaded region in the diagram is bounded by the curve $y = \frac{5}{x-2}$, the x-axis, and the lines $x = 3$ and $x = 6$. Find the volume of the solid of revolution formed when the shaded region is rotated about the x-axis. **3** Solution



NESA 2008 Mathematics HSC Examination

07 MX 1 **3a** Find the volume of the solid of revolution formed when the region bounded by the curve $y = \dfrac{1}{\sqrt{9 + x^2}}$, the x-axis, the y-axis and the line $x = 3$, is rotated about the x-axis. **3** Solution

NESA 2007 Mathematics Extension 1 HSC Examination

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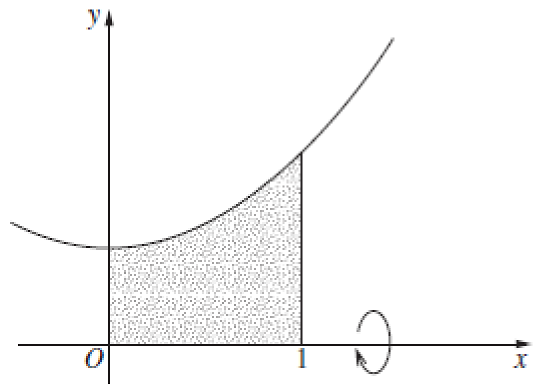
07 M **9a** In the shaded region in the diagram is bounded by the curve $y = x^2 + 1$, the x-axis, and the lines $x = 0$ and $x = 1$.
Find the volume of the solid of revolution formed when the shaded region is rotated about the x-axis. **3** Solution



NESA 2007 Mathematics HSC Examination

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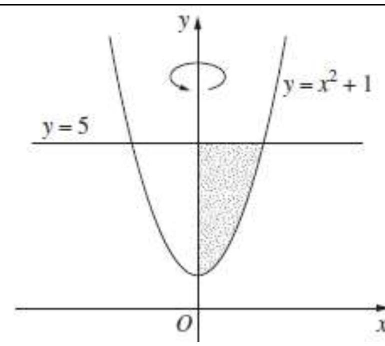
06 M **4b** In the diagram, the shaded region is bounded by the parabola $y = x^2 + 1$, the y-axis and the line $y = 5$.

Find the volume of the solid formed when the shaded region is rotated about the y-axis. **3** Solution



NESA 2006 Mathematics HSC Examination

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05 MX 1 **5a** Find the exact value of the volume of the solid of revolution formed when the region bounded by the curve $y = \sin 2x$, the x-axis and the line $x = \dfrac{\pi}{8}$ is rotated about the x-axis. **3** Solution

NESA 2005 Mathematics Extension 1 HSC Examination

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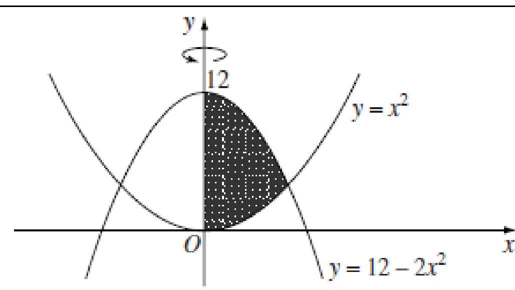
05 M **6c** The graphs of the curves $y = x^2$ and $y = 12 - 2x^2$ are shown in the diagram. Solution

(i) Find the points of intersection of the two curves. **1**

(ii) The shaded region between the curves and the y-axis is rotated about the y-axis. By splitting the shaded region into two parts, or otherwise, find the volume of the solid formed. **3**



NESA 2005 Mathematics HSC Examination

NSW GOVERNMENT

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \; r \neq 1$$

$$S = \frac{a}{1 - r}, \; |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

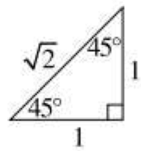
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

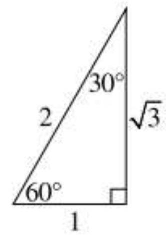
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

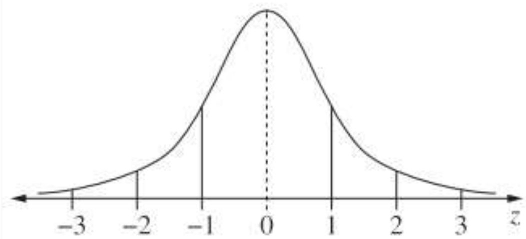
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \le x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x) = \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n\,dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$